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An Energy Circuit Language for Ecological and Social Systems: Its Physical Basis

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I. Introduction

Phenomena of the macroscopic world of ecology and sociology have been represented for many years by energy diagrams showing caloric flows. These diagrams portray the fate of energy in converging and branching networks, and suggest to varying degrees the kinetics, energy laws, and compartments of these systems (e.g., Zimmerman, 1933; MacFayden, 1963; Odum, 1957; Odum and Odum, 1959; Kormondy, 1969; and Philipson, 1966). While compartments and flows of matter, dollars, minerals, populations, etc. are frequently considered, only energy is a sufficiently common denominator to include all the forces, factors, and units of complex systems of man and nature.

In several previous papers describing complex systems (Odum 1967a, b; 1968), a symbolic language was introduced which combined energy laws, principles of kinetics, and some philosophical tenets of electrical systems. The language consists of a dozen basic modules, each having a mathematical definition. The symbols representing these modules are shown in Fig. 1. This chapter describes this energy systems language and its use. Many basic principles and equations of physics and chemistry will be cited to show their incorporation into the language

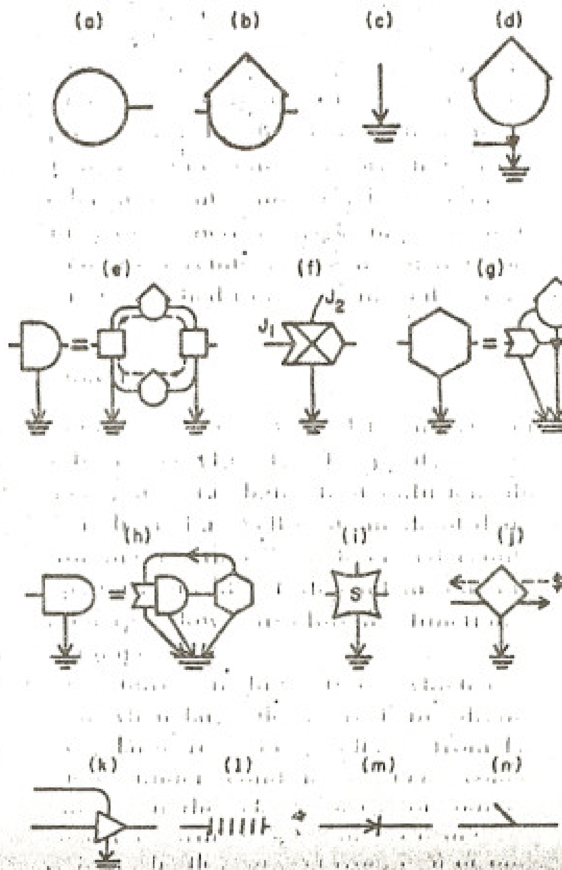

FIG. 1. Symbols of the energy circuit language. Each symbol is discussed in a [illegible] section of the text. (a) source, (b) passive storage, (c) heat sink, (d) [illegible] generating work, (e) cycling receptor, (f) work gate, (g) [illegible], (h) [illegible] plate, (i) switch, [illegible] (m) [illegible] valve, and (n) [illegible]

and its basis in them. The additive nature of energies of all types is the basis for a single network representing all processes. For a book account of energy language written for the general reader, see Odum (1970). The language is offered as an interface language for simulation.

Energy Diagram

Illustrated in Fig. 2 is an energy diagram for a house. Potential energy is derived from outside sources that have characteristic programs of delivery. It flows through the system along indicated flow lines and ultimately passes out as dispersed heat (heat sink symbols). Along the way energy may be temporarily stored as potential energy, may do various forms of work, may loop back upstream for action from a downstream position, or may interact with other energy flows. When energy transformers are connected by pathways, simple systems are

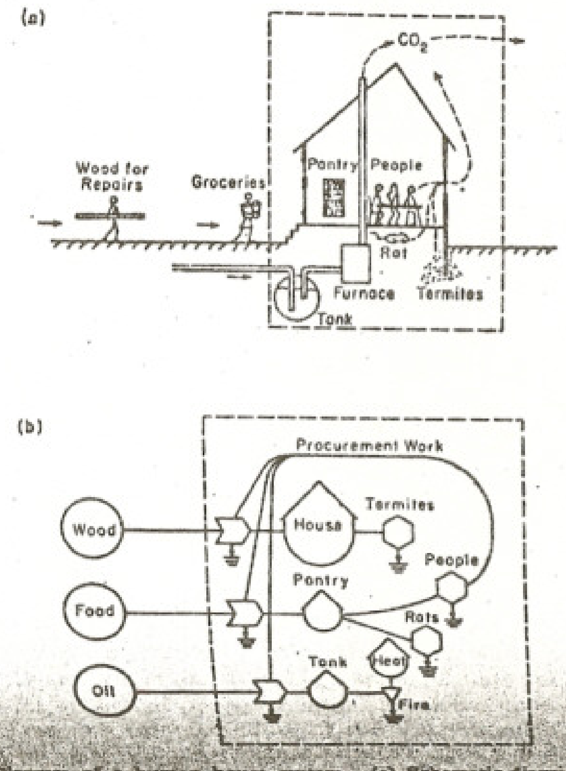

FIG. 2. Diagram of a human house system. (a) Schematic diagram, (b) energy diagram.

formed, some of which occur frequently and are basic building blocks for more complex networks (e.g., Figs. 1e, g, and h). In a qualitative and superficial way, the energy diagram helps to keep track of all possible flows and parts of complex systems, and aids in computing budgets. Beyond this, however, a degree of rigor derives from the fact that implicit in the energy circuit language are many rules and constraints derived from long-established principles of physics, chemistry, and biology.

The basic types of circuits can be found in many fields of science where the relationships have often been rediscovered independently. Rashevsky (1960) considers chains of physiological processes involving reactions and diffusion. Levenspiel (1962) summarizes chains of processes in chemical reaction engineering. Goodwin (1963) develops parallel, feedback, and more complex circuits to account for gene control of enzyme processes in biochemistry. There are many texts on electrical network elements. The same network units are considered by different writers under different names such as bionics, cybernetics, network theory, and reaction kinetics. By placing some of these concepts in the language of energy circuits and generalizing somewhat, basic combinations also may be applied to environmental systems and society.

II. Energy Source Module

Energy sources from outside the defined boundaries of a system of interest are indicated by a circle (Fig. 1a). To specify a source completely, the nature of the energy flow (as light, heat diffusion, flow of organic matter, etc.) must be indicated as well as its mode of delivery. In many systems languages "forcing functions" are described as outside programs affecting temporal patterns inside. Independent outside patterns of force delivery and energy flows are forcing functions and require independent energy storages.

One class of energy sources includes those which exert a constant driving tendency even when large flows are being drained. Examples are pressure from very large reservoirs, voltages from large batteries, and chemical reactions under conditions where concentrations of reactants are maintained. Another class consists of sources which have a constant flow of energy regardless of drains to using systems. Water flowing past a waterwheel, the flow of fuel from coal or oil fields under some conditions, and light flux are examples.

Whether a source is of constant force, constant flux output, or follows some other program, its representation is incomplete until defined, e.g., by some graphical function or explanatory equation.

Energy sources may deliver energy in [illegible] and water waves, or they may deliver energy with matter flow as with a flow of fuel or water. Sometimes one energy source pumps in a second source by doing work on its flow. The energy source symbol has one or more flow lines representing the pathway of potential energy delivery and also the pathway of action of driving forces which may be part of the energy delivery.

Another kind of energy source has a repeating pattern of energy delivery. Examples include sine waves, square waves, impulse functions, etc. A source may be stochastic, resembling noise and describable by statistical parameters.

III. Force and the Energy Pathway

Rooted deeply in human cultural origins are concepts of causal force and energy reflecting early qualitative recognition of fundamental laws. The formalization of qualitative concepts into quantitative definitions of force and power of mechanics in nineteenth-century physics did not eliminate the variety of causal force and energy concepts in everyday life, but merely added rigor to part of the area of previous application. For example, the numerical product of force and velocity is power in Newtonian mechanics, but political causes and cultural energies are still discussed in vague terms.

With the development of concrete formulas for driving tendency in various areas of evolving science such as electricity and thermodynamics, a generalized quantitative concept of force emerged. In considering various phenomena of the environmental systems, we can extend further the concepts of generalized force as a driving impetus and power as a measure of its delivery rate. Denbigh (1952) and Prigogine (1955) summarized the generalized concept of force as applied in physical science (see examples in Table I). Jammer (1957) gives the earlier history of the force concept and its use in such areas as theology and literature where its application is still qualitative.

In this chapter two kinds of force laws are identified in energy transformations of the macroscopic environment (Fig. 3). Both are incorporated into circuit notations. Although physical forces are at the root of all cause, the phenomena of interest are often the expressions of populations of forces combined in various ways whose average effects serve as a causal function in steady state. Causal force concept is generalized for groups of processes under the name "population force." Ultimately, through understanding of the distribution of input forces, population force, and loads, one hopes to understand all flows of energy through ecological and social networks.

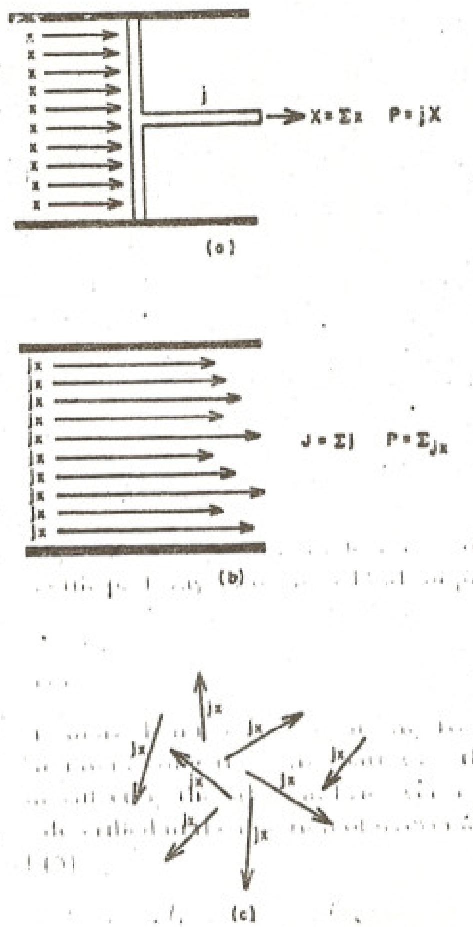

FIG. 3. Three types of organization of component power transmissions. (a) Component forces add to form a single force that determines flux; (b) component fluxes add in parallel delivery of power because of channelling; flux depends on population force; (c) component processes without organization disperse heat.

A. The Energy Pathway

In the energy circuit symbols, solid lines represent flows of potential energy from a source. The source may be an outside program of energy input, indicated by the source module, or it may be derived from a potential energy storage unit within the system (indicated by the tank symbol). Potential energy sources deliver their energy flows by directing a force. The line is the pathway of the force from the energy source.

An opposing force is directed backwards along the line in opposition to the driving force. The opposing force may be from a downstream storage, from friction of the pathway, or from inertial backforce if acceleration is involved. The pathway satisfies the equal and opposite force requirement of classical physics. The energy pathway therefore is actually an equation in which the forces are set equal at every point. It is also a pathway of flow of an energy quantity that does not increase or decrease, but only changes in form. The constraint of the first law of thermodynamics is recognized by providing that all inflows be balanced by storages and outflows.

In electrical networks, the ordinary wires are instantaneously in steady state between the force of the voltage and the backforce of the frictional resistance to the passage of electrons, but the modules that the wires connect may not be in steady state. This property is adopted for the energy network language. The isolated pathway line is always in steady state, although the system of pathways and modules may not be. The energy language by this device, as in electrical networks, has transient phenomena such as growth and decay of storages while the flow lines remain in a steady state balance of driving and frictional forces. The pathways are defined as the balance between force X from the energy source and the frictional backforces RJ, where backforce is proportional to the flux (of matter or energy, J),

$$X = RJ, \tag{1}$$

and R is the constant of proportionality (called resistance in electric circuits). Equation (1) rearranged shows flux proportional to net driving force, which is Ohm's law in the electrical case and more generally the basic force–flux law of irreversible thermodynamics,

$$J = \frac{1}{R} X, \tag{2}$$

$$J = LX, \tag{3}$$

where L is the conductivity describing frictional backforce tendencies of the pathway. The absence of a line indicates absence of a pathway. All pathways involve some kind of structure, temporary or permanent, for which potential energy must be spent for maintenance. Each pathway requires a work expenditure to maintain the upstream, downstream, or some lateral module. Each pathway shows the path of potential energy crossing energy barriers passing against frictional dissipation, or doing work against opposing forces of downstream storages. The

requirement that potential energy be dispersed as unusable heat is indicated by the heat ground diversion (heat sink symbol, Fig. 1c) of some potential energy by whichever modules are responsible for the particular pathway.

In practice, the heat sink energy diversions are drawn at one end or the other of many circuits. For example, a fish swims to obtain food and establishes by his movement a pathway of food to him, but the energy is shown in the diagram as a heat sink on his main module since energy costs are mainly derived and distributed there.

The Newtonian convention that every force has an equal and opposite force is represented by the energy pathway as a balance of forces. If acceleration occurs, it is considered within one of the modules as an energy storage rather than within the energy flow line. The energy dissipated in a pathway includes loss of potential energy in overcoming friction plus loss in specific work processes designed to maintain organized pathways as pipes, wires, spending channels, organisms, food chains, etc.

In Table I are given some of the single forces that may be driving energy flows along the pathways. For these kinds of processes Eqs. (1)–(3) are descriptive.

B. Coupled Forces

When there are more than two forces entering from energy sources or storages along or intersecting the same pathway, then there is a force component of one affecting the other and vice versa. Forces are coupled. This situation is described in the notation of irreversible thermodynamics by Eqs. (4) and (5).

$$J_1 = L_{11}X_1 + L_{21}X_2 \tag{4}$$

$$J_2 = L_{12}X_1 + L_{22}X_2 . \tag{5}$$

It is the Onsager reciprocity principle that the component L_{12} of force X_1 on the flow of the second J_2 is equal to the component L_{21} of the second force X_2 affecting the first flow J_1,

$$L_{12} = L_{21} . \tag{6}$$

This principle may be an extension of the Newtonian principle of equality of opposing forces. Thus X_1 and X_2 may be directed in the same direction along an energy pathway, in opposing directions along the pathway, or one may cross the other, doing work on it as described later for the work gate module.

TABLE I

FORCES AND FLUXES FROM POTENTIAL ENERGY SOURCES

Type of energy	Force[a] (calories per unit displacement)	Flux (displacement per time)
Symbol	X	J
Electrical	emf (V)	Current (A); Charge per time
Mechanical	Newtonian force	Velocity (d/t)
Rotary motion	Torque	Angular velocity
Heat gradient	$\Delta T/T$	Calories per time
Hydrostatic	Pressure	Compression rate (v/t)
Surface energy	Surface tension	Area per time
Light	Radiation pressure (cal/volume)	Volume per time
Molecular diffusion	Gradient of chemical potential	Molecules per time
Individual molecular chemical reaction in two parts[b]	X	J
(1) Energy activation crossing force barrier	Deceleration of momentum	Molecular velocity
(2) Chemical combination	Intermolecular and interatomic forces	Molecular velocity
Steady state population of chemical reaction processes. Two conventions, neither have both $JX = P$ and $J = LX$		
A. Free energy convention[c]	Chemical potential, μ	Rate of reaction da/dt
B. Population of forces acting separately[d]	N, proportional to number of component reaction pathways	Number of reactions per time

[a] Potential is defined as the ratio of the power delivered to the accompanying flux of matter or pure energy. Many of the forces in this column are potentials by this definition.

[b] Single molecular process involving accelerations-decelerations and thus not a steady state until considered as a population.

[c] JX is power but J is not proportional to X. This has been used in chemical thermodynamics erroneously on the justification of an approximation.

[d] J is proportional to X, but JX is not power.

Fluxes J on the energy diagram change along the routes, being material in one segment, but pure energy such as heat and light in others. The inclusion of more than one kind of energy makes energy networks differ from material network diagrams, such as those for dollars or the nitrogen cycle, where by definition there is only one kind of item flowing.

Whereas forces represented by vectors in Euclidean space have a magnitude and a direction, force vectors implied or drawn along the energy flow lines are in topological space and have only a positive or a negative direction, a force from the left and from outside sources being taken as positive. Inputs and outputs are the pertinent features in most networks.

C. Dissipative Forces

In Table II are given some forces which serve only against driving forces. The first three are frictional resisting forces. The last three are populations of resistive forces associated with some energy flows of more complex systems. They act in reverse to energy sources although they may consist of more than one process and resistive force. Some of these forces are necessary and may be useful only because they permit a low cost steady state. Others may be directly useful. A grindstone is an example of dissipative work which is useful to its system.

TABLE II

FORCES AND FLUXES IN DISSIPATIVE WORK FUNCTIONS[a]

Type of work	Force (calories per unit displacement)	Flux (displacement per time)
Frictional processes		
Electrical resistance	Back emf	Current (A)
Mechanical resistance	Friction	Velocity
Rotary resistance	Braking torque	Angular momentum
Structural controlling processes		
Pattern changing	Start-stopping force of spatial patterning per unit mass	Mass arranged per time
Organizational connections of units	Calories per circuit connection	Connections between units established per time
Maintenance	Calories per gram replacement	Rates of replacements and reorganization; (g/time)

[a] For these forces the flux J is always against the force X and does not exist without an outside potential energy inflow.

As listed in Table II, there are several kinds of dissipative work concerned with change of structure in which potential energy is passed into heat with changes in structure resulting. As separated in Table II these flows of energy can be thought of as displacements against structural forces, which like friction oppose the process. The forces exert more back resistance as the speed of the process is increased.

With *pattern-changing work* objects are rearranged. If rearranging automobiles in a parking lot, it takes the coupling of free energy to start them rolling and then when they are stopped, this energy of momentum goes out as heat in the stopping friction of the brake. If one rearranges the automobiles rapidly, one uses more energy flux for the job than if one does it slowly. As the time used approaches infinity, the energy costs approach zero, neglecting friction of the machinery. Thus, the amount of potential free energy that must be expended in pattern-changing work for speed tax is a function of the rate of rearranging. The final product involves a new state of arrangement, but no appreciable storage of the energy that went through the system to accomplish the work. Pattern-changing work as here defined, like friction, is a function of the path, not the initial and final states alone. However, there may be a path that is optimal for maximum accomplishment.

Another kind of structural work is *organizational work.* This work involves making connections between parts to form a system, thus eliminating uncertainty in relationships and decreasing the possible operation alternatives of the whole system.

Maintenance work (Fig. 1g) is a form of structural work in which some parts are replaced, rearranged, and organized so as to balance exactly the dissipative tendencies for loss of necessary system structure.

D. Energy Flow in Structure Arranging Processes with Acceleration

An important process in macroscopic systems necessary to their survival involves spatial arrangements of parts. These involve acceleration–deceleration actions and friction. The energy flows can be visualized for a simple limiting case, which is found to be a type of dissipative energy transformation.

Consider the successive arranging of identical weights in space without ordinary friction. A force is exerted on each weight, accelerating it as it moves toward its final position where it is stopped abruptly, the kinetic energy going into heat. As soon as one weight stops the force accelerates a second so that the process is steady, but pulsing. Such a system is a straightforward problem in mechanics. The input force is balanced on average by the resistive force of the stopping-heat dispersals, so that the process can be regarded in the long range as a steady-state

process. This is something like friction, except that the input force first does inertial work and then at a later time the kinetic energy goes 100% into heat. For a practical example visualize the arranging of trucks in a parking lot, arranging troops in a field, or collecting materials for construction. These examples also have ordinary friction involved although the correction is not necessarily large. The energy per parcel is proportional to the square of the average velocity. The total power is the product of energy per parcel times flux of parcels. The overall power dissipation is thus proportional to the cube of flux.

In Fig. 4c is plotted the relation of power and velocity for the arranging process showing the much increased energy flux with increased speed when the input force is increased successively. Transformation is to inertial work, which then goes into heat. The entire flow can be useful and at the same time is speed tax.

A comparison of curves in Fig. 4c shows the advantage of a steady frictional load in an arranging process as compared to an accelerative and stop type. For the same flux achieved, the power requirement as the cube is vastly greater with accelerative arranging. Friction is a very useful property to complex systems, as the astronauts found.

E. Generalized Force–Flux Concepts of Steady-State Energetics

The energy circuit language represents steady state thermodynamics, which concerns inflows and fates of various kinds of potential energy in spontaneous processes. For each kind of process whether chemical, mechanical, thermal, magnetic, electrical, etc., each potential energy flow is expressed as the product JX of the two quantities which have the dimensions of power P: J, the rate of flow (flux) of the material concerned; and X, one of the energetic forces. The J's and X's are chosen so that the product JX is power and so that each J is proportional to an X affecting it with L, the conductivity, a constant of proportionality. For example, the power entering from an energy source given in Table I is

$$JX = P \tag{7}$$

[Eqs. (1)–(6)]. (See Table I for forces and fluxes which fit this definition.) Some of the flows of potential energy are positive, representing power added to the system by input forces from potential energy sources. Others represent power delivered in overcoming backforces in the process of storing or exporting potential energy. As required by the second energy principle (degradation law) some of the inflowing power becomes a flow of irreversible heat, often termed "speed tax." A heat dispersal symbol (Fig. 1c) is defined for this flow. As required by the first energy principle (conservation law), the sum of the positive and negative flows of potential energy JX is the speed tax (P_t in Eq. (8) and Fig. 4b),

$$P_t = J_1X_1 + J_2X_2. \tag{8}$$

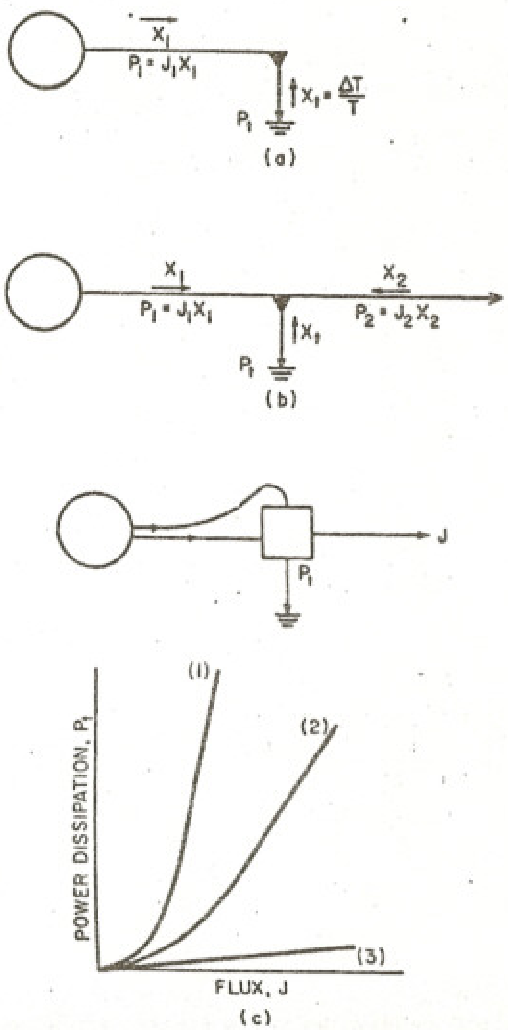

FIG. 4. Identification of forces, flux, and power flows in dissipational energy process. (a) Input power equals entropy-generating speed tax; (b) input power exceeds speed tax; (c) power dissipation of three classes of transport pathways: (1) dissipation in proportion to cube of flux, as in mechanical acceleration followed by abrupt deceleration ($P = kJ^3$); (2) dissipation in proportion to the square of flux, as in electrical flow in wires or swimming fish ($P = RJ^2$); (3) dissipation in proportion to flux, as in sliding friction ($P = kJ$).

The potential energy lost is equal to the heat dispersed in the environment by the first energy principle. This heat dispersed by diffusion processes is in proportion to the heat gradient ($\Delta T/T$), one of the thermodynamic forces (Table I). Defining calorie flow as the flux J_t of the speed tax, the power dispersed is the product of the force and flux, $J_t\,\Delta T/T$. By definition, the quotient of the heat dispersed, ΔH, divided by the absolute temperature T is the entropy, ΔS, generated in the environment, $\Delta H/T$. This entropy is a measure of the randomness produced from more ordered molecular patterns by statistical motions of molecular energy. The second energy principle requires that there be heat dispersal and entropy increase in the environment for any process to go spontaneously. Stating the principle in another way: the process must be probable. The speed tax power dispersal may be expressed in terms of entropy generated as in Eq. (9),

$$P_t = J_t\,\Delta T/T = T\,dS/(dt). \tag{9}$$

F. FORCE AND FLUX WITH ENERGY IN PACKETS

The discrete inflows of energy in such unitized forms as photons, sound waves, water waves, cannonballs, electrical pulses, nuclear particles, and molecules of organic matter are a familiar part of many systems. The number of energy packets transformed depends on the number caught by the energy receptor, which depends on the density of the packets in that vicinity. The input of power from a process of catching such energy flows is the product of the energy packet density in the receptor vicinity and the volume of that vicinity swept per time. One may identify a force and flux for such flows by choosing the density of energy packets as the force and the volume of the receptor vicinity swept as the flux. Then the flux is proportional to the force and the product of the force and flux has the dimensions of power. For example this convention has been used for light in Table I. The density of light is sometimes called "radiation pressure."

If a system is catching all of the packets incoming, then the power input is identical to the passing flow of packets. In other instances the passing energy is a reservoir of energy whose density encourages the capture of some but not all of the packets as input flux J. As long as there is incident flow in the system vicinity, it is self-renewing and acts as much as a reservoir of constant force as the infinite water table or electric battery. Energy passing without capture is unused potential.

G. Force and Flux in Diffusion of Molecules

A common area of application of the force–flux concept is diffusion. Fick's law states that the rate of diffusion flux of chemical molecules, J, is proportional to the gradient of concentration ([ ]).

$$J = k\, d[\,]/d\delta, \tag{10}$$

where δ is distance. Since pressure is a function of the concentration of molecules, the chemical potential energy (free energy per mole, μ) can be rewritten as Eq. (11) (μ refers to one molecular species; ΔF (see below) refers to all substances present),

$$\mu = RT \ln [\,] \tag{11}$$

[See Eqs. (14) and (31)–(33)]. Taking the derivative of (11) with respect to distance δ, one finds

$$\frac{d\mu}{d\delta} = \frac{RT}{[\,]}\frac{d[\,]}{d\delta}. \tag{12}$$

Combining (10) and (12), one writes the expression for diffusion in terms of the gradient of chemical potential,

$$J = \frac{k[\,]}{RT}\frac{d\mu}{d\delta}. \tag{13}$$

If the flux of molecules is taken as J and the gradient in chemical potential as the force X, JX is power and J is proportional to X, consistent with other forces and fluxes in Table I. For an example of application of these concepts, see Best and Hearon (1960). While chemical potential is a force in diffusion, it may not be in chemical reactions.

H. First Convention for Choosing Force and Flux in Chemical Reaction Systems

There are special problems in identifying force and flux in chemical systems so that the flux J is proportional to the force X [Eq. (3)] and the product of force and flux JX is power flow [Eq. (7)]. In chemical thermodynamics one convention, as given in Table IA, is to choose the chemical potential energy (ΔF or μ, free energy per mole) or its negative, the affinity, as the force, and the reaction rate as the flux. When this is done, Eq. (7) holds, but flux is proportional to potential only in special cases. Chemical reactions go at speeds proportional to

concentrations, not in proportion to free energy available. As derived in Eqs. (31)–(33), the potential energy (constant pressure) per mole is proportional to the *logarithm* of the concentrations of reactants and products of the process concerned:

$$\Delta F = \Delta F_0 + RT \ln \frac{\text{products}}{\text{reactants}} \tag{14}$$

(ΔF_0 is for the standard state).

If the chemical potential ΔF is the logarithm of the concentration, the flux, rate of reaction J, is proportional to the concentration; then the relation between flux and chemical potential is tested by combining Eqs. (14) and (3), letting X be concentration,

$$J = \frac{1}{R} e^{\Delta F/RT}, \tag{15}$$

where R is the chemical resistance. The flux is not proportional to ΔF and hence ΔF is not correct as a choice of force.

The reasons that there is difficulty in choosing force and flux in chemical systems is that one is not dealing with a single process but with a statistical sum of separate physical processes. As considered subsequently there may be a more accurate causal concept than chemical potential for statistical assemblages.

I. Semantic Problem of Linearity

Many processes of the macroscopic world when expressed in the form of Eq. (1) are not linear, i.e., R is not a constant. However, some of these processes are actually compound linear circuits by which energies are involved in feedbacks and other arrangements. It is a basic aspect of the energy language that there is a linearity in the simple single pathway, although the compound modules which these simple pathways form may be nonlinear. It is customary in many fields such as electrical engineering to define the linear process as one in which the flux is proportional to upstream storage, referring to other processes as nonlinear, even though these other processes are made up of components that are all linear. For those in other fields this custom is sometimes a source of semantic misunderstanding. A radio with many pathways each of which is following linear processes has many modules, whose overall equations are nonlinear. *For steady-state thermodynamics and its macroscopic application here using a concept of a linear population force, all complex nonlinear processes may be dissected into linear* component pathways. In this sense all processes are linear and all processes are in proportion to a force, either a single force or a population force. Compound modules are nonlinear, but their nonlinearity is a result of the configurations of more than one linear pathway.

For example, in micrometeorology and plant physiology the diffusion of carbon dioxide in a forest is often represented with Eq. (1); R is described as a parameter that varies with place, height, and time of day. It is not really a constant but a variable because the process is not a simple pathway; R is actually being misused. In energy language there is a multiplicative intersection of energy flow from an energy source in turbulent eddy flux. The component processes are linear, but the combination is not. Whereas the use of variables as parameters has been an objective process, the energy circuit language has a more basic ideal of breaking compound processes into their linear component circuits or if compound modules are used, they should have clear definitions in relation to elementary units and with the corresponding differential equations.

IV. Population Force

The energy circuit language recognizes two levels of force, only one of which has been discussed in the previous paragraphs. The classical physical concept of a single force, such as a force of gravity, an electromotive force, a magnetic field force, etc., is represented by the energy pathways as the line of action of these forces, and the movement of energy along the pathway is in response to these forces. As frictional backforce adjusts to equal the driving force [Eqs. (1)–(3)] a balance is achieved by which the flux is proportional to the force. Work is done. For these regularly recognized physical forces, the energy circuit language is suitable for showing the network of actions, pathways, and energy sources. These forces may be made up of smaller component forces, as when water is additive in the water tank or electrical cells are additive in voltage in the series battery, but the output pathway has one resultant force, where the component forces are additive.

The energy circuit language also recognizes another level of force which is pertinent to the population sciences, defining population sciences as those in which the phenomena of interest result from parallel actions of a population of small forces. Chemical reactions, biological processes, sociological phenomena for example, are mainly concerned with the sum of a population of separate actions. Whereas people can pull together to develop one force on a battering ram, their usual actions are to work in parallel each contributing to a group output.

For these types of energy flows, each separate action has a force and flux of a regular physical type (previous paragraph), but the *flux of output of the group is proportional to the number* of simultaneously acting units. Flux is proportional to the number of forces (population force), which is designated N. Whereas the energy per component action is as previously discussed, the total energy is proportional to the number of actions. The variable is N, not X.

The energy circuit language where populations are involved expresses the action of N, the population force, in driving the flow of energy bearing flux J,

$$J = LN, \tag{16}$$

which is as fundamental a proposition as the force–flux relationship of the single force [Eq. (3)]. Both types of force operate in the energy circuits, one compounded of the other. Both are linear propositions, and hence the energy circuit language has as its basic feature a linearity of flow in response to a force from an energy source or storage. In some cases the line represents a single force X (where its component forces are adding) and in other instances the line represents a sum of parallel drives, the component forces being according to the population force expression N.

A. Components of Population Force

Many processes in nature combine the action of populations of separate events that individually are one-way actions not in steady state. The overall changes in chemical reactions are the sum of actions of populations of individual molecules; food chain flows are the sum of individual food flows in populations of organisms; the flow of fuel through the transportation industry is a sum of flows through populations of motorized units. For each component event we can identify a true force x and flux j as in Table I for single chemical events. We can then add the component forces and component fluxes to obtain a group number for each quantity, but if we do, the product of the sum of fluxes and sum of forces is not the sum of the power that one obtains by summing the total of the individually separate jx products. Arithmetically, the sum of products is different from the product of the sum. It is simply not mathematically *possible to represent the population of chemical reaction processes by a flux proportional to a force whose product is power although one can do this for each individual reaction.* Erroneous early misapplication of steady-state thermodynamics to chemical reactions resulted from this error (see correction by Denbigh, 1952.) The erroneous choice of force and flux is given in Table IA. In order to distinguish the components from the population quantities we designate small letters for the component force x, component flux j, and component power jx. We use X' as the sum of the component forces and J' as the sum of the component fluxes. The following derivation [Eqs. (17)–(22)] is made for populations of equal components, a special case of a population of unequal components.

For the separate component processes we rewrite Eqs. (3) and (7) with the component symbols (small letters) including l as conductivity of the component process,

$$j = lx, \tag{17}$$

$$P = jx = lx^2. \tag{18}$$

When the component processes are acting separately, there are numerous parallel flows whose total flux is given in Eq. (19) by multiplying the flux per unit by the number of components n and by the probability p of their operation at any one time:

$$J' = pnj. \tag{19}$$

Combining Eqs. (19) and (17) the flux resulting from the components is given in terms of the component properties:

$$J' = pnlx. \tag{20}$$

For a population process in which p, l, and x are constant, the population flux J' is proportional to the number of units n, which may thus be regarded as a linear causal factor. In the chemical case x is chemical potential, Δu (Table I).

In general both the mass q and the number n as measures of quantity of component fuel units acting in parallel are in proportion so that the flow may be written as a linear function of either if appropriate constants are used. Because the equations follow closely some analogous ones for electrical analogs, it is convenient to introduce the quantity N as population force, defining it in terms of number and mass as in Eq. (21). Population force is proportional to the quantity of matter driving the population flow, but differs numerically from n or q according to the constants of the system involved:

$$N = Q/C = nq/C. \tag{21}$$

The ratio of mass Q to population force N is the constant C analogous to electrical capacitance, and q the weight per individual unit enumerated by n. See also Eq. (34).

Combining Eqs. (20) and (21) and using the constant L' to represent the group of constants $plCx/q$, one may write the causal equation of population flux as a function of the population force in a form analogous to that for the force–flux equation:

$$J' = (plCx/q)\,N = L'N. \tag{22}$$

Thus we define a causal factor N for a population phenomenon using a different convention (B in Table I) from that often used in chemical systems. Many chemical, biochemical, biological, ecological, and industrial systems can be studied with this expression in which flux is proportional to the cause N. However, the product of population force N and flux J' is not power. The quantity N was previously discussed as "ecoforce" (Odum 1960; 1967a), but since it applies to chemistry and cellular biology as well as ecosystems and sociology and is not a single force, it now seems better to use a different name, "population force."

In the special case in which the potential energy W is in proportion to the number of component processes (as when grain is stored in an elevator), the flux in Eq. (22) is also proportional to potential energy.

B. Proportionality of Potential Energy, Potential, Force, and Power

Sites of potential energy storage (symbols in Fig. 1b and d) have potential and exert force. Potential is the work done per unit moved from equilibrium. Force as defined broadly here is that function to which the flux is proportional and may be a single force or population force. In some systems such as electrical ones, the flux of amperage is proportional to the force of voltage, which turns out also to be a potential so that flux is proportional to potential. With chemical reactions, flux is proportional to the population force, a concentration gradient, but not to potential. Chemical potential is a logarithmic function of concentration ratios. However, other systems in which flow is proportional to population force in the macroscopic dimensions, transmit packages of energy.

The energy storage is made up of these packages of energy set so that the flow is proportional to the population of packages and can at the same time be proportional to the potential energy storage and the potential. Thus, many ecological and social systems operating with these relationships are simpler than the electrical and molecular ones. In energy circuit language, the pathways and storages represent energy storages and flows. The flows are proportional to the type of force prevailing (single force or population force). Potentials, storage functions, and relationship of flux and power to these vary with the type of energy storage and flow and must be specified if this becomes of interest.

In the energy circuit language the pathways are always flows of power. Flux, however, is variously defined in different segments of the network. The flux may be defined as identical with energy flow in heat and other pure energy flows. In other systems it may be the flow of a material such as electrons or water (Table I). In some systems, those that fit the group of thermodynamic forces and fluxes in Table I, the product of single force and flux is power. In other population flows the power in the pathway is not the product of population force and flux. It is the product of the energy per component μ and rate of flow of components,

$$P = \mu J = \mu L'N. \tag{23}$$

C. Power and Forces Delivered by Populations in Series and Parallel

The power and force delivered by a population of separate duplicate processes depends on the manner of combining the components (Fig. 3). If the flows are parallel, the fluxes add; and if the components are joined the forces add. A familiar example is a population of electrical dry cells. In either case the total power for the population of processes is the sum of the powers of the components jx:

$$P = \sum jx = pnjx. \tag{24}$$

Note in Eq. (24) that the power is proportional to the population size n as well as to the magnitudes of component force and flux.

In processes such as radioactive decay and metabolism of populations of people, the flows are separate, the forces act separately, and Eqs. (19) and (24) can be combined as (25):

$$P = J'x. \tag{25}$$

The flux J' is the sum of the components, $\sum j$, and the force is that of each component.

In other processes, such as living electrochemical cells uniting electrical outputs in series in an electric eel or people exerting pull on the same rope to combine forces with one flux and one group force, Eq. (24) can be expressed in terms of a single group force X' substituted for pnx:

$$P = jX'. \tag{26}$$

There is one flux j that goes through higher resistance and the force X is the sum of components $\sum x$. The regular definitions of force and flux in Eqs. (3) and (7) are appropriate.

When power flow in chemical systems that follow Eq. (25) (for separate action) is divided by the flux J, according to the procedure for calculating potentials, the component force x results. This quotient, power per unit flux, is μ, chemical potential. The erroneous convention A in Table I for selecting force gives the component force rather than the variable quantity that determines rate of flow, because in this class of chemical reactions the potential function [Eq. (27)] is not linear with concentration. Convention B is more appropriate for selecting cause where populations are in parallel action.

Populations of chemical molecules can act either in series or with forces united depending on the process. When there is a chemical reaction where molecules are reacting by collisions in many separate actions the forces are separate and convention B is appropriate, since the rate is proportional to the population force N [Eq. (22)]. However, when a surface area is provided against which the molecular moments may sum to produce a pressure or through which they may thrust an osmotic pressure, the forces add, and the rate is proportional to the group force X' [Eq. (26)].

When an electrode is provided and equilibrium achieved (potential energies equal), a third type of force expression results because electrons derived from the population of molecular reactions at the surface act together to develop an electrical potential E that is equal to the chemical potential of the solution ΔF as given in Eq. (27). In this expression, f is Faraday's constant and n the number of moles. Due to the self-repulsing charged nature of electrons their potential is the same as the electromotive force. The force thus expressed when the electrode and solution are at equilibrium is equal to the potential and hence to the logarithm of the number of components. When the electrode is connected so as to flow electrons externally there is no equilibrium and the flow may then follow Eq. (26). Thus, the type of force provided by a population of component forces depends on the circuit connections of their action.

Figure 3 diagrams the parallel and combined force arrangements for two processes with the same overall power output. Both can be represented by the same energy circuit:

$$nfE = \Delta F = \Delta F_0 + RT \ln \frac{\text{products}}{\text{reactants}}. \tag{27}$$

Where the forces combine there is one force and the flux passes along one route. Where the forces are acting separately, we combine them in our energy study by placing them in a compartment because they are indistinguishable and for the larger system are of little separate interest. Hence we can represent the population as one group flux J' and identify the force involved as the component force x, while using the population force N as the linear causal factor of importance affecting the flow. In Fig. 3c the forces are not channeled but dispersed in stochastic manner, interfering with the opportunity of the parts to accomplish group action. Such a system is normally nonadaptive and is excluded by selection unless such disordering serves some role, as it sometimes does. Examples of dispersive actions which are adaptive are: urban renewal, dispersal of seeds, and distribution of rain.

Summarizing, the pathway of the energy language always represents the flow of energy from a potential energy source, a balance of forces, and a steady state in that segment when its inputs and outputs are steady. The nature of the causal force is not automatically defined by the pathway line. One must add vectors or other auxiliary notation (i.e., Fig. 3) to specify whether a given segment is a single Newtonian force proportional to potential of the driving source or the causal linear drive of a population force proportional to number of component driving process elements. When diagramming and analyzing simple systems such as electrical networks and groundwater, the Newtonian drives are involved [Eq. (2)]; when diagramming and analyzing ecological and social systems, population force drives predominate [Eq. (16)]. Both can be used in the same energy network system. One may quantitatively diagram flows and storages without necessarily knowing which applies if power data are available.

In analyzing systems of energy pathways and their modules differential equations can be written following either of two procedures: (1) The rate of change of a storage in terms of input and output energy flows can be written substituting expressions for each flow in terms of other source and storage forces. (2) Forces that converge on a point can be set equal, and expressions substituted for each force in terms of sources and flows. These procedures are employed in commentary that follows on equations and behavior of each module in terms of the internal networks they represent.

V. Heat Sink Module

The second energy principle requires that all spontaneous processes include dispersal of potential energy as heat P_t distributed into the environment, unavailable further as a driving impetus for processes.

Wandering of molecules makes the process go and by definition disperses its molecular motions collectively called "heat." The heat sink symbol (Fig. 1c) represents this energy dispersal, which must occur spontaneously from energy operating module of an energy system. When some simple system such as a water flow or electrical resistance is being represented, the heat sink receives flows directly from the pathway (Fig. 4). In complex modules that represent groups of processes, the heat sink is a miscellaneous conduit of heat dispersal of the many processes, grouped together for convenience. The respiration of an organism is the sum of the many processes of work and heat dispersal. Respiration is easily measureable by the oxygen consumption or carbon dioxide production which accompanies these processes, and measurement of total flow to the heat sink is a good starting point in evaluating the energy flows of complex systems. Whenever of interest, particular component processes may be isolated, labeled, measured, and represented by separate flow lines and heat sinks, leaving by subtraction, a lesser flux of heat dispersal represented by the main miscellaneous heat flow of the module (Fig. 1c).

As shown in Fig. 4, heat dispersal into the heat sink can be visualized as a flow from a source of potential energy which exerts a driving force in the form of a thermal gradient and is opposed by a frictional force inherent in the property of entropy increase (X_t in Fig. 4). See Eq. (9).

The heat sink symbol represents the last stages of potential energy dispersion when molecular and thermal diffusion is dispersing any potential energy remaining in molecular component populations. When heat is dispersed by eddies and other self-generating thermal engines, an outpumping subsystem exists that requires identification, diagramming, and evaluation. The symbol for this is more than the simple heat drain. Additional modules with energy storage and pumping work are required, as described for Fig. 1g.

VI. Passive Energy Storage Module

When potential energy is stored within the defined system it is represented by the tank symbols (Fig. 1b and d). Energy storages can exert true forces either together as a single Newtonian force or in parallel as a group population force. A potential energy storage is not completely defined until the quantity of potential energy in calories is expressed and its storage form defined by an appropriate equation. Water stored in a tank against gravity has energy and force both related to its hydrostatic head; energy in compressed gas is related to pressure by a logarithmic function. Energy and population force delivered from a silo of stored grain is mainly proportional to the number of grains. Potential energy is stored by doing work against the resisting force of the potential storage just as one stores potential energy when one presses a coiled spring against its resisting force. The more work that is done against the backforce of the storage unit, the more energy there is stored and the more force that unit can deliver when a pathway of outflow is provided.

In Fig. 1b passive storage is indicated by which the potential energy developed by work elsewhere is moved into a storage location without creating new potential energy. Such a flow requires an energy diversion or a second energy flow to move the potential energy into position, doing work against friction and energy barriers, but not increasing the quantity of potential energy by moving it into situation. Pumping gasoline, stacking potatoes in a store, or placing dynamite in an explosive shed are examples.

When an energy storage reservoir consists of a collection of component identical energy-containing packages which are not exerting forces on each other, the potential energy of that reservoir is the product of the mass of storage units Q and the energy per unit mass μ. This is the population potential energy W:

$$W = \mu Q. \tag{28}$$

For example, a bin of potatoes constitutes a group of units each with chemical potential energy, the total potential being the product in Eq. (28). If one were computing the potential energy within each potato in terms of the molecular composition one would use a different formulation for the potential in terms of storage forces overcome during synthesis. For macroscopic processes, however, the potential energies of storage reservoirs are given in proportion to the number of storage units present. The energy of storage may be delivered from the site as packages, or it may be provided with a pathway that expresses its component forces so as to discharge the energies stored within each package.

Storage Functions

Following are some of the storage functions by which energy may be stored against backforces of the storing process. A storage module is not adequately described unless its storage function is indicated. In the passive module, the storage work is done before the potential energy is brought into the module in packages.

Where the addition of successive stored units does not involve changing forces x or displacements h against which the work is done, the potential energy is proportional to their number n:

$$W = nxh. \tag{29}$$

Elevation of successive equal weights to a platform against gravity involves a constant gravitational force and the same vertical distance of displacement. Storing of compounds in a chemical industry is another example, as is the reproduction of offspring of equal size which may serve as potential energy to other food chains.

Where the back force of the potential is increasing as a function of the energy units as they are stored, summation requires integration. In pumping water into a vertical cylinder, the force (gravity g) against which work is done increases as the weight of the accumulated water increases with height. Water density is s, and area is a:

$$W = a \int sgh\, dh = \tfrac{1}{2} asgh^2. \tag{30}$$

Where the force of the potential is exerted due to the combined actions of accumulating molecules behaving as ideal gases in a gas phase or a dilute solution, the sum of the work done is the sum of the product of compressed volumes and pressures overcome with pressure and volume responding inversely according to the gas law,

$$Pv = nRT; \tag{31}$$

$$W = \int P\, dV = \int \frac{nRT}{V}\, dV = nRT \ln \frac{V_1}{V_2}, \tag{32}$$

$$W = \int V\, dP = \int \frac{nRT}{P}\, dP = nRT \ln \frac{P_1}{P_2}. \tag{33}$$

Because of the reciprocal relation of pressure and volume or any molecular concentration and its group thrust across a plane surface, increase of molecular concentration adds less and less potential energy in the logarithmic relation of Eq. (33). This applies to pumping gas molecules into a tank, to the thrusting of molecules through a membrane, or to the packing of electrons from an electrode into a solution reaction. Thus the group force of molecules on a plane surface is in proportion to their number, but the accumulated potential energy stored there is in proportion to the logarithm of these numbers. The water vapor potentials existing between various components of the soil such as roots, clay, and organic matter is not in proportion to the vapor pressure, but logarithmically related as in Eq. (33).

The change in potential energy (excluding that required to change volumes while adapting to constant pressure) was also called "free energy, ΔF." Equation (33) is the expression for free energy change in terms of molecular concentrations or pressures. Free energy is appropriately used where processes, as in the biosphere, are under a constant pressure to which any gases appearing adjust volumes until they have equal pressure. Work of such volume adjustments is omitted when free energy is calculated, but has to be included in a complete energy budget.

In storage of electrical charges (Q) on condensers with electron flow in metallic conductors, the charges have electrostatic self-repelling field forces (voltage $E = Q/C$) so that the force overcome in adding an electron to storage is proportional to charges already accumulated:

$$W = \frac{1}{C}\int_0^Q Q\,dQ = \frac{Q^2}{2C}. \tag{34}$$

VII. Potential-Generating Storage Module

When the work of generating potential energy is done within the system it is represented with the modular symbol of Fig. 1d, which shows heat dispersal into the heat sink accompanying the pumping of potential energy into storage against backforces of that storage. Depending on the kind of work being done, the nature of the storage function may differ and a full description of the system requires an expression of this. The potential generating process involves at least three forces: the driving input force, the backforce from the potential energy storage being increased, and the frictional backforces of heat dispersal (Fig. 5). When the input and potential generating forces are equal, the system does nothing and is defined as a thermodynamically reversible one that can go either way with slight addition of energy to either of the two balanced forces. The second energy law requires that for spontaneous movement there be some heat dispersal and thus for the input force to exceed the potential generating backforce (Fig. 5). As described previously (Odum and Pinkerton, 1955; Tribus, 1961), selection for maximum rate of power storage adjusts the backforce to be half that of the input force. The maximum power property is implied by the modular symbol whenever power selection has been attained as theory predicts for real self maintaining systems. Where useful power expenditure is storage, maximum power selection of a one-step process leads to load ratios and storage efficiencies of the middle point in Fig. 5. The non-steady-state Atwood's machine has maximum power efficiency higher than 50%.

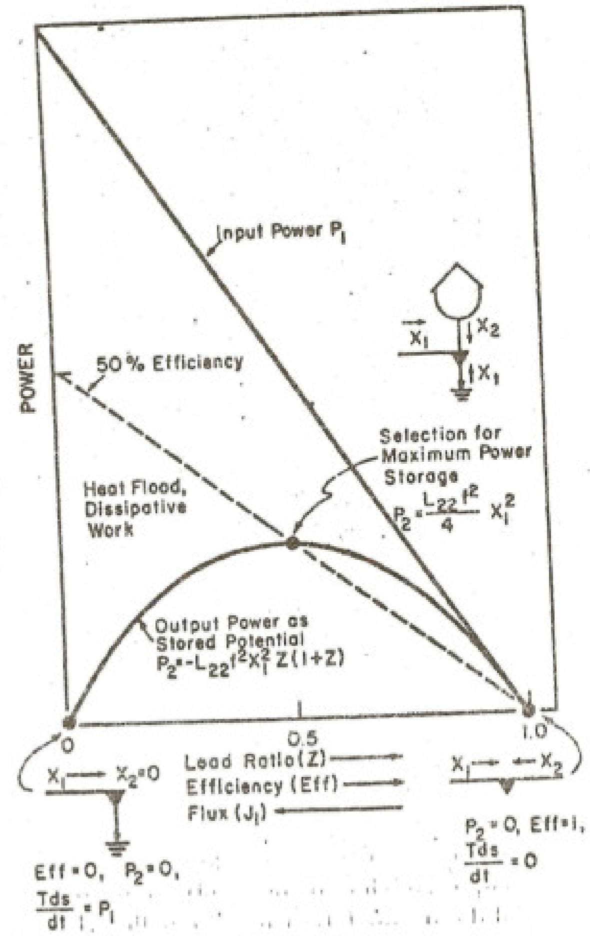

FIG. 5. Power input and useful power output as a function of load ratio, efficiency, and input flux, when input force X_1 is constant and output load X_1 is varied.

When provision for storage of materials and energy is inserted in a flow so that there is an opportunity for generation of potential energy W characteristic of the storage, then the potential energy of storage exerts a force depending on the potential force against which the work is done.

When two different kinds of storages are connected so that one can transfer energy to the other, unequal storage site forces will shift energies until equilibrium is attained. The potentials are then equal. For example, metallic electrical systems can be connected with chemical processes in solutions and gravity potentials can be connected with gas pressure systems in hydraulic systems.

A. One-Way Valves

Adopting the diode symbol (Fig. 1m) for a valve that permits flow in only one direction, the property of undirectional flow may be indicated in the energy circuit diagrams. Many of the flows have this property because outside work is required to traverse pathways but energy for backflow is unavailable. The diode symbol need not be drawn if the module on the end of the pathway has the one-way property as part of its interior structure. The self-maintaining module, for example, has work gate feedbacks controlling inflow and thus has little backflow. The value does allow backforce to affect flow.

B. Linear Decays of Storage

In Eq. (35) P represents the power delivered by potential energy source W which declines in potential energy dW/dt by the power delivered:

$$P = dW/dt = u\, dQ/dt. \tag{35}$$

In simple potential energy storages, decline in potential energy W may be accompanied by decline in output force of that storage X, but in compound population storages, it is the number of energy storage units that declines and the force that declines is the causal group action of the population N. The decay of energy follows the flux in this way only when the energy storage function is also in proportion to the quantity of units stored Q. An electrical capacitor is an example of one that is not. The mass of charge on an electrical condenser discharges with pattern of Eqs. (35)–(37), but not its energy, which is proportional to the square of the charge stored [Eq. (34)].

One of the well-established experimental confirmations of the linearity of many complex population storages is the linear decay when inputs to storage are eliminated. The decay rates of radioactive populations, of sewage solutions, of soil litter, or starving animals, and many others have as first approximation flow dependent on storage, the outflow being equal to the rate of decrease:

$$dQ/dt = J = L'N = (L/C)Q = (1/RC)Q. \tag{36}$$

Here, L/C is the fractional turnover rate and C/L the time constant. The integrated form of Eq. (36) is

$$\frac{Q}{Q_0} = \exp - \frac{L}{C} t. \tag{37}$$

Energy storages W follow Q in those population examples with linear storage functions only [Eq. (28)].

The constant L/C may be evaluated from a semilog graph of mass with time.

C. Storage Kinetics

The energy storage has characteristic Von Bertalanffy kinetics as it achieves a steady state when an inflow J is supplied. Equations describing the storage with time are similar for three cases: (1) the energy storage may be passive but filling faster than its outflow which depends on amount stored; (2) in filling a storage in a potential-generating situation, increasing potential is developed against increasing static back force; (3) the driving force is accelerating matter or accelerating electrical flow so that potential energy is developed in the inertia of mass or the electrical-magnetic field which opposes any acceleration until it ceases, after which the energy of the inertia or of field that has been stored drives the current. The latter property is used in electrical circuits with wire coils as inductances to add time lags that are out of phase with those achieved by filling a tank against static potential generating forces. This impeding property may be generalized in complex living systems by behavior programs which have the same property of resisting input while storing energy in proportion to the original impetus, later diverting the energy storages into actions that do later what the original force might have done had it not been blocked. (A symbol is given in Fig. 11.)

Whatever the process, storage rate is a balance of inflows and outflows. Sometimes referred to as the Von Bertalanffy equation, the storage tank is appropriately called a "Von Bertalanffy module" with saturation equations (38) and (39). Energy is proportional to mass Q only in those systems with linear storage functions as in many population examples (see Fig. 6):

$$dQ/dt = J - L'N = J - (L'Q/C). \tag{38}$$

If initial storage is zero, the integrated equation takes the form

$$Q = \frac{CJ}{L}\left(1 - \exp - \frac{L}{C} t\right). \tag{39}$$

This is the charge equation of electrical circuits where C/L is the time constant.

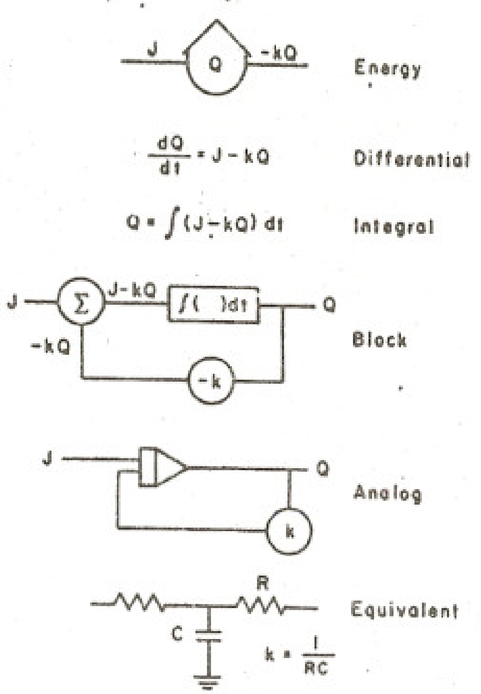

FIG. 6. Von Bertalanffy storage module stated in six languages.

D. EFFICIENCY IN POTENTIAL-GENERATING CIRCUITS

The loading of backforces from an output process which stores potential energy (Fig. 4b) is in a different class from passive dissipative loads. A potential load has an independent value that is not dependent on the flux but is determined by its own energy storage level or by an external potential in the case of an export circuit. The amount of loading can vary from zero to a stopping load which converts the process into a reversible stationary situation. The load may be greater than the input force so that the process reverses with energy flow and flux in the opposite direction.

Unlike the dissipative flows, power output against a potential load is not a conversion of potential energy into dispersed heat and thus by the second energy principle will not take place unless there is at the same time arrangement for some diversion of heat flow for speed tax. Thus the speed with which power output is directed into a potential circuit depends on the amount of speed tax, other things being equal. The efficiency of power transmission into the new potential reservoir from

the old [illegible] given in Eq. (40) by the ratio of expressions for power given in Eq. (8):

$$\text{Eff} = \frac{P_2}{P_1} = \frac{J_2 X_2}{J_1 X_1} = \frac{J_1 X_1 - P_t}{J_1 X_1}. \tag{40}$$

Classes of power output in potential generating circuits are those given in Eqs. (28)–(34). An output of heat which develops a reservoir of heat at a temperature higher than the surroundings is potential energy, but an output of heat which disperses without changing temperature gradients is dissipative and serves as speed tax.

In order to measure the potential force loading, let us define a ratio Z as the quotient of the potential generating force X_2 and the input force X_1, inserting f to place both forces in the same dimensions so that Z is dimensionless:

$$Z = X_2/fX_1. \tag{41}$$

When the output potential force X_2 balances the input force X_1 the process is stalled as on the right in Fig. 5. Then the force ratio Z is one, the flux J_2 is zero, and Eq. (4) may be combined with (41) and rearranged to provide:

$$f = \frac{X_2}{X_1} = \frac{L_{12}}{L_{22}} = \frac{L_{11}}{L_{21}}. \tag{42}$$

The ratio of conductivities f can then be evaluated. Energy flows vary with load ratios Z (Fig. 5). Any leakage is regarded as a separate circuit.

E. ENERGY FLOW IN LEVER TRANSFORMATIONS

As in elementary science books, levers such as the inclined plane, screws, pulleys, gear wheels, lever arms, etc., can transmit power with little loss while transforming the relative magnitudes of flux and force reciprocally. Thus, in Eq. (7) the force X may be varied with a compensatory change in the flux J so that power expressed is unchanged except for losses to heat dispersal. An example of a lever transformation is change from situation (a) to (b) in Fig. 3.

The concept of torque for the force on a rotary system is usually taught as the product of linear force and the radius, similar products exerting similar rotary accelerations, even though force and radius vary. The torque concept is derivable from the concept of power conservation during a lever transformation. Since the velocity of a point on a wheel is proportional to the radius, the compensatory change of J and X is equivalent to a compensatory change [illegible]

F. The Possibilities of Loading

The backforce against which a process does work is the loading. These load forces determine the partition of power budget among several flows and whether there is optimal loading of each flow (Fig. 5).

If the work is one of storing energy, it is potential generating, and according to the second energy principle if the potential-generating output load equals the input load, no power flows. Otherwise there could be use and reuse of energy. The size of the potential-generating load affects the speed and power transformed through that flow. It affects the energy restored as compared to that dispersed as speed tax,

$$J_1X_1 + J_2X_2 = P_t = P_1 + P_2, \tag{43}$$

where the fluxes are related to the forces as in Eqs. (4) and (5). Substituting (4) and (5) into (43) and also the definition of force ratio Z from Eq. (41), we obtain

$$P_2 = -L_{11}X_1^2Z(1 + Z). \tag{44}$$

With forces to the right considered positive X_2 is taken as negative, and Z is negative. This convention is different than that used previously (Odum and Pinkerton, 1955) in order to simplify application to the energy circuit diagrams. Equation (44) expresses the useful power output in terms of the loading ratios of the output to input forces Z, where the input force X_1 is constant and L_{11} is the input conductivity.

If one sets the derivative equal to zero so as to find the value with maximum output, one obtains $Z = -\frac{1}{2}$ and the equation for maximum power is

$$P_{2\,(\max,\,X_1\ \text{const})} = (L_{11}/4)\, X_1^2. \tag{45}$$

Substituting (4), (42), and (44) into Eq. (40) for efficiency of restoration, one obtains Z for the efficiency and 0.5 for the efficiency at maximum power output:

$$\text{Eff}_{(\text{power max})} = P_2/P_1 = -Z = 0.5. \tag{46}$$

Evaluation of constants in a particular problem can be made with data in particular cases. With output force set to balance input force so that the process stops, the ratio f is evaluated from Eq. (42). If while stalled there is a flux J_1, it occurs through some other energy flow circuit which has not been stalled by backforce. One may either include such flows in the equation as a leakage term, as done previously (Odum and Pinkerton, 1955), or exclude them from consideration as a separate

circuit, as done here. Later the overall input and output efficiencies can be computed for a network if there are several diverging flows. Hence, in describing the power-efficiency relations of a single circuit, J_1 is zero when Z is one. The conductivities L_{22} and L_{11} as previously discussed may be regarded as the flux-proportional coefficients of resistive force with X_2 or X_1 as input forces, and the conductivities may be evaluated with flux measurements in situations with one force input against resistive drag.

Thus L_{22} is equal to the coefficient of a resistive force when a force X_2 is imposed on the system while X_1 is zero and flow is in steady state. The other conductivities are related to L_{22} and are evaluated as follows: When the output force has stalled the input force as in Eq. (41), f is computed relating dimensions of L_{12} to L_{21}. With X_2 zero and considering all leaks as part of other circuits, L_{11} is the coefficient of resistive force after a flow from X_1 is in steady state through the system. When the input flow is stalled with backforce X_2 so that J_1 is zero, then the conductivities L_{11} and L_{21} are related as their dimensions f in Eq. (41) and summarized in Eq. (42). Some other conversions among conductivities may be written as

$$L_{11} = L_{22}f^2 = L_{21}f. \tag{47}$$

Equations (44) and (45) are plotted in Fig. 5. The point of maximum power may be chosen by natural selection under the conditions discussed in which the power is flexibly usable as specific competitive criteria require. Selection of the maximum power point also sets the force ratio and hence the efficiency at 50% [Eq. (46)]. Although one might theoretically load a system so that the force ratio (and efficiency) was at some other value, it would not be a competitive arrangement where power storage was useful. The loading would either be too slow or waste too much energy as speed tax. One may state this as a fourth energy principle that natural selection adjusts the speed tax and efficiencies of potential output to 50% for a single transformation. Whereas power going into heat was found to be linearly proportional to flux when the input force was constant in single dissipative flows, the speed tax to heat on a potential conversion goes up as the square of the flux. It is the area between curves in Fig. [illegible]

G. Power Output When Potential Loading is Constant but the Input Force is Varied

The relationship of output power with varying load and constant input force was summarized as a parabolic relation in Fig. 5. Equally important is the pattern of power output that results when the input force is varied while the output potential load force is held constant (Fig. 7). Increasing the input voltage on a battery charging process is an example.

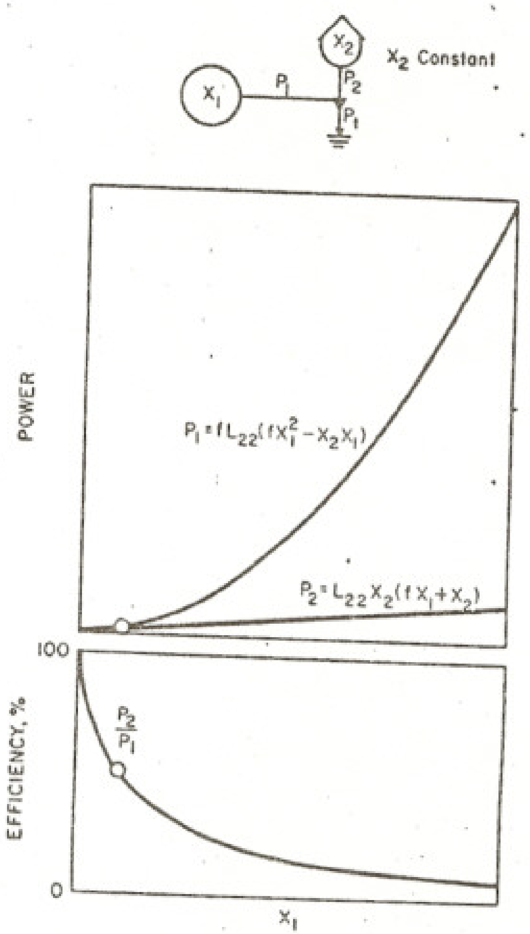

FIG. 7. Power storage P_2, efficiency of power storage, and input power P_1 when output loading X_2 is constant and input force varied. Loading of 50% is encircled.

In a procedure similar to that used for derivation of Eq. (45), an expression for power output against the fixed potential load X_2 is developed as a function of the variable input force X_1. The result is a linear equation,

$$P_2 = L_{22}X_2(fX_1 + X_2). \tag{48}$$

which is graphed in Fig. 7. The power input P_1 varies as the square of the input force,

$$P_1 = fL_{22}(fX_1^2 - X_1X_2), \tag{49}$$

and is also graphed in Fig. 7. The ratio of power flows is the efficiency, in Eq. (50) as a function of the load ratio Z, defined in Eq. (41):

$$\text{Eff} = \frac{P_2}{P_1} = Z. \tag{50}$$

This efficiency is graphed also in Fig. 7. When f is one, the efficiency is the force ratio. From the steeply declining hyperbolic pattern of the efficiency plotted as a function of the increasing input force X_1, it can be observed that under increasing input force the system is using greater and greater quantities of input power, and greater and greater amounts are going for speed tax. Beyond the point of optimum efficiency marked with a circle in Fig. 7, the system is very uncompetitive.

If a system of varying inputs were set up so simply, it would rarely function effectively. It will be shown that biological systems with varying inputs have various devices for maintaining a more optimum loading ratio by varying the output loads and by other means. Like automobiles they have their gears changed as the input force is increased. Photosynthesis is one example.

Because some properties of speed, starting characteristics, and other aspects are selected by engineers in the design of such engines as electric motors, there results the same kind of selection for power as exists in the natural selective process. In both kinds of systems there are various devices for regulation, starting, and control so that other loadings are not allowed. It is thus not always possible to vary load experimentally in order to observe the changes in percentages of speed tax, efficiency, etc. in the basic speed regulator curves (Fig. 5).

H. Resistance Matching of Dissipative Processes in Flux Series

When there are two dissipative processes whose fluxes are in series, such as two resistances in an electrical circuit, the one with the greater resistance draws the greater proportion of the power dissipation of the two, but it is also the rate-limiting process. If the larger one diminishes its resistance, it then allows the flux through both units to increase and hence its own power increases, but its proportion of the total power then diminishes. When the two resistances are equal, the power is maximal relative to other combinations with the same total resistance. If one resistance is lowered below the other it [illegible] drain, but the proportion shifts to the other so that the smaller one has less power dissipation than when it was equal.

A familiar example of dissipative processes in flux series is the battery with external resistive circuit. The battery acts as if it has a certain internal resistance. Maximum power is drained to the outside when the resistance outside is made identical. A higher resistance slows down the flow too much. A lower one dumps energy within the battery.

The relation of power and load on the second of a pair of dissipative transformations in series is similar to that of power and potential conversion load following the hump shape of Fig. 5. However, selective processes need not necessarily favor maximization of a resistive dissipation. If the second of the two processes is a useful one, selection processes may act to regulate its speed and thus its loading. If the second process is not by itself a useful one it may serve a useful function to the first process if that one is useful, regulating its power flow. If selection causes a system's loading to be set so that the second process is at maximum power output, this makes the first process subject indirectly to speed control by maximum power selection. A single dissipative process has no maximum in its relation of load to power, but the double series system with input force held constant does have a power-loading curve with a maximum.

I. Power and Efficiency of Populations of Processes Each with Constant Force Loading

Equations (39)–(46) related power and efficiency where forces were varying. Somewhat analogous equations may be written for systems where the individual component forces are acting separately, are not varying, and have each backforce opposite an input force. The total flux, power, and efficiency are then the effect of population force and population backforce, the numbers of actions being the variable. Input forces n_1 are more numerous than opposition forces n_2. The ratio is $z = n_2/n_1$.

An input system with n_1 separate input actions may have some of its population of input forces x_1 opposed by n_2 negative backforces x_2, whereas others may be opposed by no backforce other than resistive forces implied in the component conductivity l_{11}. The number of unopposed inflows is $n_1 - n_2$. The opposed and unopposed flows are given in Eqs. (51) and (52):

$$J'_{1\,(\text{unopp})} = l_{11}x_1(n_1 - n_2). \tag{51}$$

$$J'_{(\text{opp by } n_2)} = (l_{11}x_1 + l_{21}x_2)\, n_2. \tag{52}$$

By adding opposed and unopposed populations the total input flux J_1' results:

$$J_1' = l_{11}x_1n_1 + l_{21}x_2n_2. \tag{53}$$

Since n_1 is greater than n_2, there are no unopposed component forces x_2 and we may write for the output flux

$$J_2' = (l_{12}x_1 + l_{22}x_2)\, n_2. \tag{54}$$

For the simple case of all component loads ratios represented by one Z value, expressions for power delivery follow from Eq. (18) with substitution of expressions (41), (42), and (47) for the group flux J':

$$P_2 = J_2'x_2 = -l_{11}x_1^2n_1zZ(1 + Z), \tag{55}$$

$$P_1 = J_1'x_1 = l_{11}x_1^2n_1(1 + zZ). \tag{56}$$

Where the component forces, fluxes, and loading ratios are constant, the power delivered is proportional to the ratio of populations z opposed as well as the population of input forces n_1. The input flow is proportional to the size of the population of actions n_1 diminished by the ratio of opposing thrusts, where z and Z are both negative. The ratio of the power flows is the efficiency:

$$\text{Eff} = \frac{P_2}{P_1} = \frac{-zZ(1 + Z)}{1 + zZ}. \tag{57}$$

Increases in the ratio z of opposed forces of a population of useful outputs result in increases in flux, power, and efficiency without maxima up to a value of $z = 1$. In Eq. (57), for example, efficiency is maximum when the force ratio of the component processes Z is 0.5 and when the ratio of number of processes z loaded is 1 (100% loaded). The overall efficiency is the 50% for maximum power storage possible.

VIII. Intersection and Feedback

The intersection of two flows involves phenomena that depend on the kind of reaction. If the two flows are of similar materials and energy types, they may add forces and flows as in the examples of two converging water pipes or copper electrical conductors. Such *additive junctions* are shown in our notation (Fig. 1n) as simple intersections that may exist without any heat losses. If there are no one-way gates, the flows may subtract forces and reverse the flows that [illegible]

If, however, an intersection involves different kinds of flows that react in definite proportions, the output may depend on the product of the two driving forces and may be called a *multiplicative junction* (Fig. 1f). The intersection may have a constant effect, drawing power as needed (Fig. 1k) as a constant gain *amplifier*. Figure 1i is an intersection operating switching action of one flow on another. In the notations for environmental networks the kind of functions involved at each junction must be indicated by an appropriate symbol.

If a flow of energy or materials has a circuit connecting a down-circuit section to an up-circuit section so as to form a circular loop, it is called "a feedback circuit." For example, there is a feedback of electrical power from the generator to the water pump in Fig. 8f. The connection of the feedback with the main flow forms a converging junction, but the closed loop causes the loop system to exhibit characteristic properties that are different from intersecting junctions without feedback.

There are many kinds of feedback. Some circuits return materials without much force or energy; others feed back both force and energy. The feedback may serve as transport and energy barrier boost without otherwise affecting the force, energy, or materials flowing. Amplifier flows may be involved in some part of the loop so that the feedback has more force and energy than the original up-circuit inflow. In other systems the feedback has a diminished force and serves as a gate on the upstream flow.

IX. Constant Gain Amplifier Module

If one energy flow undergoes a transformation so as to increase the force and energy exerted through a second-flow by a constant factor, the combination constitutes a constant gain amplifier. The interaction of the two energy flows is drawn with the symbols shown in Fig. 1k. The combination follows the second energy principle by dispersing more potential energy into heat in the amplifier flow than is stored in the flow amplified.

If either the increased force X_2 or the population force N_2 after amplification is proportional to the input force, one may represent the change with Eq. (58) where g is the amplification factor often referred to as gain. The triangular amplifier symbol of Fig. 1k implies that power is drawn as needed to make the Eqs. (58) and (59) hold:

$$X_{out} = gX_{in}, \tag{58}$$

$$N_{out} = gN_{in}. \tag{59}$$

There are other amplification functions also. The symbol g may be constant or variable. If the gain is high, the down-circuit process may be guided by the up-circuit process without drawing much power from the input.

A. Feedbacks That Add or Subtract

Shown in Fig. 8 are some feedbacks that pass the same type of force and energy flow back to an upstream site so that they add or subtract. Examples are electrical circuits that feed back electrons and voltage to upstream junctions, or fluid pipes that return downstream fluids to upstream sites in chemical reaction industries. The loops closed by these circuits may have outside energy sources and amplifier actions as shown in Fig. 9d.

Feedbacks may be classified according to the nature of the feedback intersection as additive or multiplicative and positive or negative. The feedback may be a pathway of flow of force and energy as shown in energy circuit language in Fig. 8. Also there are feedbacks of effect without a special pathway. Negative feedback is implied in the Von Bertalanffy storage module in which the outflow is proportional to the storage. In Fig. 6 the energy storage symbol with a steady inflow J and density dependent outflow kN is shown. The differential equation for the storage is a balance of the two flows. Converting to integral equations and diagramming the terms in block language, a classical negative feedback, stabilized system results. In other words any storage unit is a negative feedback stabilized integrator of energy flows even though there are no actual pathways of force or material feedback. The property in ecology of density-dependent population regulation turns out therefore to be classical negative feedback. Also shown in Fig. 6 is the electrical analog equivalent of one storage compartment. Much ecological and tracer simulation so far has involved chains of these units.

B. Feedback Loops Which Add Forces

One may characterize the general pattern of additive (or subtracting) feedback pathways with equations relating forces and transfer functions. Using symbols in Fig. 9d, the force X_3 or population force N_3 exerted by a feedback loop is given by

$$N_3 = BN_4 \quad \text{or} \quad X_3 = BX_4, \tag{60}$$

where B is the feedback function, which may be a simple constant or more complex function and may be positive or negative. If gain is

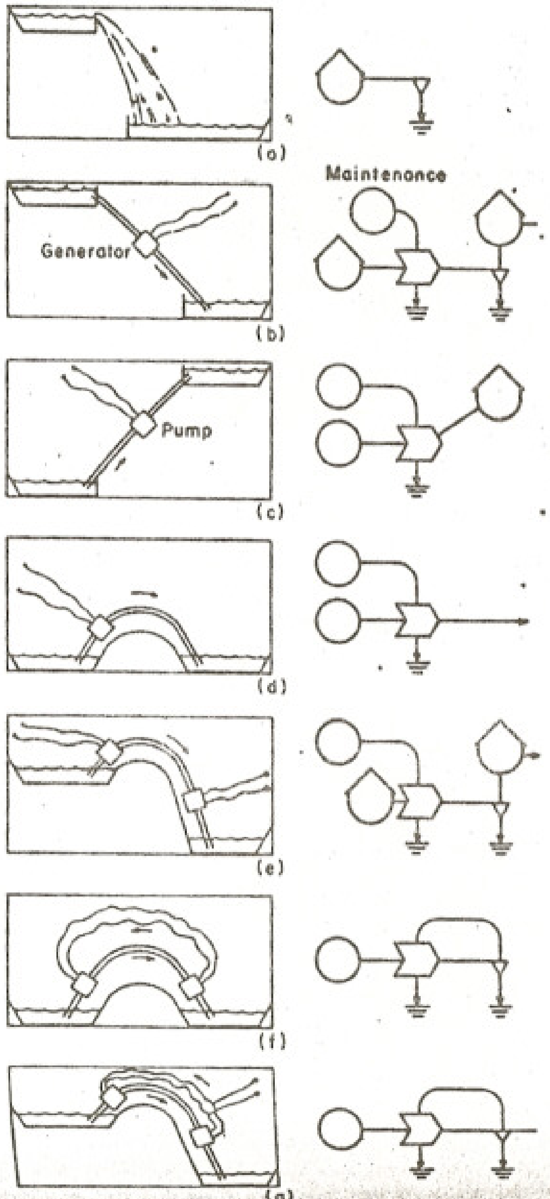

FIG. 8. Some hydrologic examples which illustrate energy transport pathways and their expression in energy circuit language.

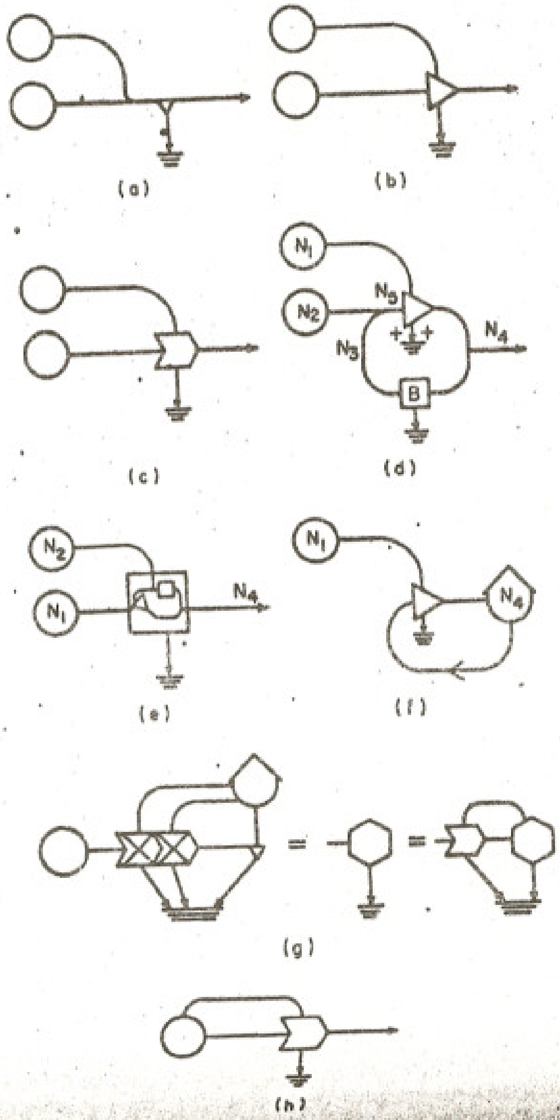

FIG. 9. Types of feedback, intersections, and multiplicative junctions. (a) Additive junction, (b) constant gain amplifier, (c) work gate, (d) and (e) additive feedback, (f) Malthusian growth, (g) logistic loops, (h) self-pumping.

involved, an outside power source may be operating (Fig. 9b). In Fig. 9d B is a box symbol (a square box is reserved as a general purpose symbol where a function is unspecified or one for which a special symbol has not been assigned). In Fig. 9e an abbreviation has been drawn from Fig. 9d emphasizing the role of force N_2 modulating a power flow from N_1.

In the circuit notation of this chapter, a constant gain amplifier stage is indicated, as in Figs. 1k and 9b, with the power supply from the top, the controlling force from the left and the feedback flow extending from the down-circuit to the up-circuit junction. The energy circuit usage differs slightly from the well-established usages in electronics. The symbol does not mean a switch in sign (plus–minus) and it must carry a heat sink and power supply, features that are not necessarily shown in diagrams of electron flows.

Stable feedback loops apply a force at the feedback junction with opposite sign so that the net force is the algebraic sum of the forces at that point, as stated in Eq. (61) using symbols shown in (Fig. 9d):

$$N_5 = N_2 + BN_4 . \tag{61}$$

Combining (61) with Eq. (57) for the transfer function of a linear amplifier, gX_5, one obtains the following basic equation for servo-mechanisms:

$$X_4 = \frac{g}{1 + gB} X_2 , \qquad N_4 = \frac{g}{1 + gB} N_2 . \tag{62}$$

The force X_4 beyond the down-circuit loop is related to the input force X_1 according to the fraction that contains the gain factor g and the feedback function B. With values of g and their algebraic sign, the steady state output of the loop may be calculated. If the loop X_2 is negative to X_1, there is a stabilized steady state. If, however, the feedback is positive, Eq. (62) provides only an instantaneous transient value in a rapid growth sequence.

When the feedback circuit acts to diminish the force, flux, or energy of the main flow, it is a negative feedback. The more one increases forces at the junction, the more feedback force there is in the role of a cancelling force, regardless of variations in the energy inputs to loop amplifiers. Whereas positive additional feedback is unstable, negative feedback provides a mechanism for stability of flows in spite of noisy variations in participating flows.

With positive feedback (Figs. 9d and e) an increase of force at the feedback junction produces an output increase at N_4 which then

[illegible] the drive at ($N_2 + BN_4$), which then increases the force at N_4, etc. Such an arrangement draws increasing amounts of power from storage or inputs within the loop, N_1, and accelerates. It is unstable, since no steady state is reached until ultimately, the acceleration ceases due to limitations in the power sources.

The Malthusian circle is the special case of positive feedback drawn as Fig. 9f, in which the output of an amplifier is fed back for amplification again with no outside flows other than the supply of power and materials entering at the amplifier as needed to provide gain. Such a system accelerates growth of materials, forces, storages, and energies.

X. Work Gate Module

Shown in Fig. 1f is the work gate module that represents the work of one flow of energy controlling and facilitating conductivity of a second. If the effect of the second flow on the first is linear, the module in effect has an output proportional to the product of the driving forces of the two pathways, and in that case has a multiplication sign × on it. By custom the main power flow is shown as the horizontal one and the limiting and controlling lesser energy flow is shown passing in from above. The heat sink represents the usual spontaneous heat increases associated with any process including those from the coupling of the two forces. The smaller flow is the signal in an amplifier effect in controlling a larger low, but the gain is a variable, proportional to the forces. The smaller control flow may come from another part of the network or from outside energy sources:

$$J = kN_1N_2 , \tag{63}$$

$$J = kN_1X_1 , \tag{64}$$

$$J = kX_1X_2 . \tag{65}$$

If one energy flow facilitates the transport of a second flow without increasing or decreasing the latter's force or energy, the first flow is accomplishing work of transport. Transport junctions are amplifiers with gain equal to the ratio of the control flow to power flow. In several natural systems it seems to have a value of the order of 100.

The transport may involve temporary amplification followed by a loss of the temporary gain. The transport may involve structural maintenance work, work of accelerations and braking, and other means for supplying auxiliary energy to pay for the energy transport of the second circuit. For convenience of recognition the transport junction is indicated by the circuit work function in Fig. 1f. If no constant amplification is involved. Amplifier and transporter junctions are given in various combinations for some hydroelectric systems in Fig. 8.

A. Crossing Energy Barriers

The existence of circuits in environmental systems often depends on auxiliary sources of potential energy that allow transport flows to cross energy barriers. For example, the maintenance flow of a repair man (Fig. 8b), permits a generator flow on a permanent basis. Without barrier-crossing transport junctions, complicated environmental networks could not exist.

According to the second energy principle a pathway proposed so as to involve an elevation of potential will not flow unless more than equal expenditures of other potential energies are provided. Thus, water does not flow uphill without a pump, and electrons will not cross a vacuum tube without an energy source to heat the filament. A circuit with a route through a state of higher potential thus has an energy barrier that requires an auxiliary energy source as a pump, although the ultimate source of auxiliary booster circuit may be the same.

In Fig. 8 are schematic sketches and energy diagrams for water pumping situations, illustrating energy barriers and energy pumps for barrier circuits. In (b) is the coupling of gravitational energy generating electrical potential energy with accompanying speed tax. In (c) is the reverse with electric potential energy generating gravitational potential in raising water with accompanying speed tax. In (d) the flow crosses an energy barrier with the combination of pumping of sketch (c) and a frictional dissipation of energy after crossing the barrier. In (e) the energy on the falling side regenerates some electrical potential with some energy for speed tax and friction. In (f) there is a feedback of the electric potential generated to the pumping flow with levels and loads so adjusted that there is no net potential. Finally (g) shows the flows of (f) but with some net generation of electrical potential.

For situations (c)–(f) as shown, there are three ways of drawing the energy circuit depending upon one's point of view. If the water flow is of principal interest, it is shown with the horizontal input line and the electrical input is shown as a work circuit flow with down-directed arrow across a circuit box.

Where there is an energy barrier not involving thereafter a net gain or loss of potential energy of the flow, the auxiliary energy source supplying energy to pass the barrier is represented as a wholly dissipative work function, as in Fig. 8d. All of the potential energy gained in crossing the hill in the example is dissipated by friction in coming back down.

In (f) there is a feedback to the barrier pump of the energy generated by the fall of water downstream. Like the somewhat simpler siphon which has the same energy diagram, the process, if interrupted, requires outside energy for restarting.

Using the terminology and diagrams of Fig. 8, one may characterize energy diagrams by the work necessary to maintain flow. Much of the energy of complex systems is dissipated in pumping other flows over energy barriers and gaps. For complete representation the diagrams should also have maintenance work flows for any structures involved.

Since the work flux of maintaining flow across energy barriers involves dissipation of potential energy into heat either from the outside or from a feedback, work flows are not reversible. Cross-barrier systems are ordinarily unidirectional. Transporting and amplifying junctions are usually one-way gates. Since the transport circuits involve the interaction of the flow transported and the work flow of the pump, they are usually double-flux reaction systems with kinetics as discussed subsequently below.

B. Diagramming Pathway Detail with Work Gates

As first presented with Figs. 3 and 4 energy flow pathways were diagrammed as simple lines plus some heat sink dispersion either along the pathway or localized in a module at one or both ends of the segment, these indicating the main association of the entropy increasing process by which the segment was spontaneously driven. The box in Fig. 4c left the functions as an unspecified operation of the input energy. This procedure can represent simple or compound processes vis-à-vis their contribution to the overall system. The single line compartmentalizes much detail of the pathway. This procedure is useful where pathway details are not of interest.

If, however, the pathway details are of interest, they may be diagrammed further by adding those modules that show the energy flows more explicitly. For example, Figs. 8f and g show details of water flow between two lakes in which the energy resources available to that pathway drive the flow through various work diversions on the pathway. These could be represented by the simple lines of Fig. 4b if the detail were not of interest.

The magnitude of an energy barrier is measured by the temporary potential energy which a unit of flux will develop in passing over the barrier. In chemical systems this potential energy quantity is the free energy of activation. In chemical systems the stochastic distribution of energies among component molecules [illegible] higher energies with enough impetus to cross energy barriers that would block other molecules. In chemical reactions repulsive forces of molecular charge often constitute energy barriers. In effect a subpopulation of the molecules is being pumped over a barrier by power transformations within the others. The energy circuit is Fig. 9h. The Arrhenius relationship of reaction rate and temperature reflects the nonlinear distribution of excessive molecular energies with temperature.

Whereas classic teaching of thermodynamics relates rates of reaction to the energy barriers and not to the potential energy of initial and final states, most real systems of the natural world develop networks in which there may be feedback of some of the potential energy of the overall reaction into maintaining the pathway or aiding the crossing of the energy barriers (Fig. 8f). The amount of energy which may be derived from the two states in this feedback arrangement is a function of the nature of the pathway. Classical thermodynamics emphasizes the equilibrium state at which there is no pathway and only the potential energy independent of path is appraised.

C. Multiplicative Junctions with Limiting Flows

If two flows intersect at a junction where they react in definite ratios and the output is the product of the two driving influences [Eqs. (63)–(65)] the processes are said to be second order. Many biochemical reactions are double flux reactions as are processes on a larger scale, such as the flows of parts to be combined in an industrial assembly line. The amplifier and energy barrier transporter transformations are special cases.

Multiplicative junctions are diagrammed in Fig. 10. Figure 10a is the energy diagram showing two converging energy flows intersecting in the work module (symbol marked ×) that represents the multiplicative effect of the separate influences at the reaction site. Also shown (Fig. 10a) are two auxiliary flows under driving influences X_5 and X_6, which serve as transport pumps in supplying the two energy flows to the reaction site developing a force due to the potential energy in the states of the reactants assembled. The rates of pumping by the auxiliary flows control the rates of supply of the reactants. At the steady state the rates of energy transformation conform to the rates of supply rather than vice versa. In Fig. 10a, if the inflows are fast the concentrations in the site storage increase and the thrust increases the reaction and/or the bypass flow. If the inflows are small, the concentrations at the site are diminished. The energetic cost of the whole process may be measured by the energetic cost of the auxiliary pump of the limiting process.

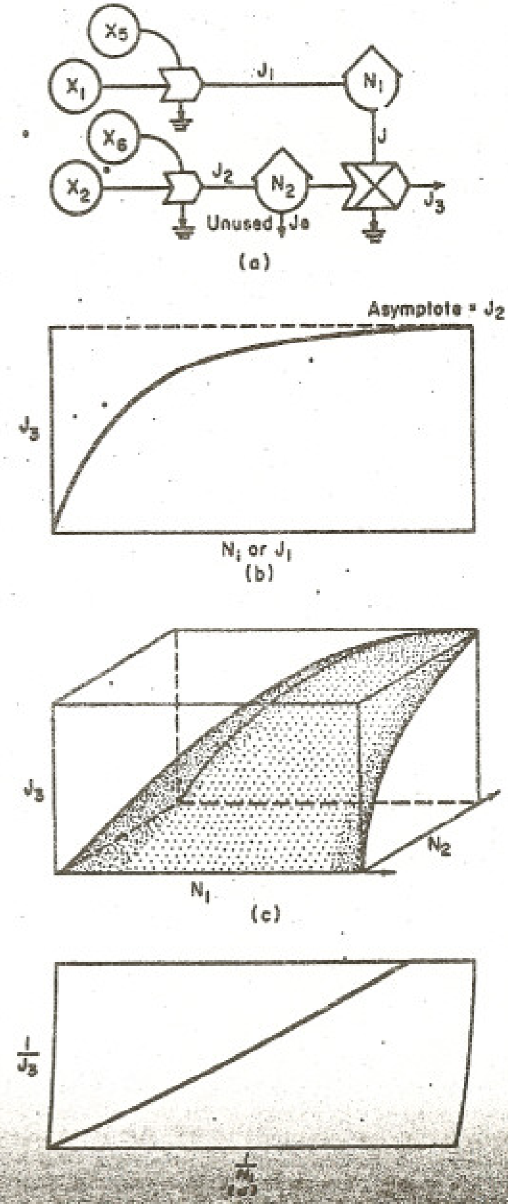

FIG. 10. [illegible]

These auxiliary costs may be derived from outside as shown in Fig. 10 or from a feedback.

In Fig. 10a is a material diagram for those examples that show the convergence of two material flows J_1 and J_2 to form one flow J_3 plus some bypass of excess as J_e. There are, however, many double-flux processes where only one of the flows contributes material to the outflow, the other inflow contributing only energy. Examples are the transport transformations that were already discussed. Figure 8c shows the simple flow of material where the second branch of the inflowing reaction involves no material contribution.

Equations (63) and (65) are familiar relations from mass action chemical kinetics for the reaction flux when the combination is in proportion to the opportunity for combinations according to the concentrations at the reaction site. A double flux process multiplies the forces of N or X.

For some macroscopic flows, N_1 and N_2 are the populations of reacting units each exerting component forces and energy transformations. For the system of flows drawn in Fig. 10 the concentrations N_1 and N_2 increase or decrease until the flux J_3 is consistent with the inflows J_1 and J_2 as driven by remote forces X_1 and X_2 and, if present, their outside-supported pump drives X_5 and X_6. Thus J_3 cannot exceed the stoichiometric requirement of the smaller of the two inflows. Changing the junction process by changing k in Eq. (63) only modifies the concentrations N_1 or N_2 necessary at the junction for steady state. Equations (66) and (67) describe material flows in the reservoirs at the reaction site in Fig. 10a:

$$dN_1/dt = J_1 - J = J_1 - sJ_3, \quad (N_1 \text{ limiting}), \tag{66}$$

where s is the stoichiometric ratio J/J_3 of necessary use, and

$$dN_2/dt = J_2 - SJ_3 - J_e. \tag{67}$$

At steady state the rates of change in Eqs. (56) and (57) become zero with Eqs. (68) and (69) resulting:

$$J_1 = sJ_3, \tag{68}$$

$$J_2 = SJ_3 + J_e. \tag{69}$$

Except for the special simple case of J_1 and J_2 in the ratio of their reaction formulas, one flow is less than the other relative to the reaction outflow and hence is limiting. If J_1 is chosen for the limiting flow, it has no excess beyond that necessary for the reaction. Outflow J_e exists

... for the reactant inflowing in excess, J_2. Equations (68) and (69) thus define the concept of limiting factors.

In double-flux flows the energy may be associated more with one flow or the other or with the relation of the two. The viewpoint as to which flow is the energy is sometimes relative. With a flow of fuel and oxygen reacting in a flame, one might regard the energy as in the fuel out of mental habit, since the fuel is ordinarily the rare quantity and the atmospheric oxygen the more commonplace. Yet with a fire within an oil mass deep in the earth one may prefer to regard the oxygen as the energy source. Actually both contribute.

D. Limiting Concentrations to Double Flux Reactions

Involving extensive experimental data in several sciences are studies of double flux reactions in which the quantity of limiting reactant at the reaction site is controlled as an independent variable. Thus N_1 in Fig. 10a is varied while the influx J_2 is in constant excess. For this special case the export of the excess J_e is proportional to the quantity N_2:

$$J_e = LN_2. \tag{70}$$

Combining (63) with (68), (69), and (70), one obtains a rectangular hyperbola, Eq. (71), relating output flux to limiting concentration:

$$J_3 = kJ_2N_1/(L + SkN_1). \tag{71}$$

The relationship is graphed in Fig. 10b. When N_1 is small, J_3 is almost a linear function of the limiting concentration at the junction site. As N_2 becomes larger, the reaction flux approaches an asymptote determined in part by J_2, the other input flux. Authors have sometimes described the curve artificially in three zones: linear, limiting-nonlinear, and nonlimiting. In this derivation N_1 was limiting. If N_2 is limiting, a similar equation results with different constants and N_2 is the variable. However, it is simpler to place the usual limiting source in the position as controller from above in Fig. 10a, swapping designations.

Expression (71) derived above from the circuits is the equation of Monod (1942) for the effect of limiting growth requirements on bacteria (Novick and Szilard, 1950). It is the relation to which many plant physiological data and agricultural experiments on plant requirements have been related. The flows of nonlimiting constituents such as J_e have not always been controlled or kept constant so that many experiments do not precisely follow the relation.

Shinozaki and Kira (1961) and Ikusima (1962) expressed, from experimental data, the limiting effects of water, carbon dioxide, nutrients, and light on plant growth as a reciprocal equation,

$$\frac{1}{J_3} = \frac{k_1}{N_1} + k_2 . \tag{72}$$

Here J_3 is the rate of gain in weight of plants, k_1 and k_2 are constants, and N_1 is the limiting factor. Rearranging this expression, the hyperbolic form is again found,

$$J_3 = N_1/(k_1 + k_2N_1). \tag{73}$$

An advantage to writing the rectangular hyperbolas such as Eq. (71) in reciprocal form, as Eq. (74) is the straight line relation which simplifies the fitting of observed data:

$$\frac{1}{J_3} = \frac{L}{kJ_2N_1} + \frac{S}{J_2}. \tag{74}$$

As in Fig. 10d, $1/J_3$ is plotted versus $1/N_1$. Ikusima (1962), for example, found the equation for duckweed population growth as a function of various limiting factors using this procedure.

Verduin (1964) discusses the Baule–Mitscherlich equation that was offered earlier to explain the asymptotic graphs from limiting factor studies. The equation is a form of the saturation equation (39) with limiting concentration substituted for time. Although the exponential graph empirically resembles the rectangular hyperbola, the theoretical pertinence has apparently not been shown.

E. Multiplicative Flows Limited by Diffusion

Consider next the special cases in which diffusion is the source pumping in the limiting reactant (no X_5 in Fig. 10a). Equation (16) may be rewritten as (75). Then N_1 exerts backforce, and X_1 in Fig. 10a becomes N_0:

$$J_1 = L_1(N_0 - N_1), \tag{75}$$

where N_0 is the concentration of the limiting material outside and N_1 the concentration at the reaction junction. If the concentration of the nonlimiting reactant N_2 is maintained constant at the junction, one may write Eq. (76) for the steady state by combining equations (63), (68), and (75):

$$J_{3(N_2\ \mathrm{const})} = \frac{L_1kN_2}{L_1 + kr_1N_2} N_0 . \tag{76}$$

Changes in the outside concentration of the limiting flow by this relation produce linear increases in the process. However, in most situations N_2 is not and cannot be maintained constant since it is used at greater rates as reaction speed increases.

If the nonlimiting reactant N_2 is being supplied at a constant rate by some mode of transport J_2, there may be an excess J_e that flows out according to Eq. (69) when J_3 is small. According to Eq. (70) this excess is removed in proportion to its concentration at the junction N_2. Combining equations we obtain (77) for the multiplicative outflow in terms of the external concentration of limiting reactant N_1 and the nonlimiting flow J_2 where $r_1 = 1/L_1$:

$$J_3 = \frac{N_0J_2}{r_1sJ_2 + SN_0 + (Le/k) - r_1SJ_3} . \tag{77}$$

Rearranging (77) as (78), one may recognize the form of Rashevsky's (1960) equation for respiration J_3 as a function of external concentration N_1 of a limiting requirement being supplied by diffusion:

$$N_0 = r_1sJ_3 + \frac{LeJ_3}{k(J_2 - sJ_3)} \tag{78}$$

Equation (79) is copied from Rashevsky (1960) for comparison without changing his symbols,

$$X = Y + \frac{Y}{1 - Y}, \tag{79}$$

where X is concentration and Y is the fraction of maximum metabolism to which the curve is asymptotic. This function like the simple hyperbola also rises to an asymptote as the outside concentration of limiting reactant N_0 is varied. At low concentrations the effect is almost linear, but at higher concentrations the asymptote is reached where J_2 is then limiting. Thus, varying the concentration of limiting reactant at the junction as in Eq. (71) produces somewhat similar results to varying the concentration at some distance so that it affects inflow as in Eq. (77).

Many special cases of double flux reactions in physiology were studied by Rashevsky, Landahl, and associates (Rashevsky, 1960). In one case flux J_2 was the flow of a fuel like glucose and J_1 was the oxygen. In another case the oxygen diffused in, but the fuel was supplied from internal storage as in some yolk-rich eggs during early embryonic development. Rashevsky's equations become more complex than the basic form in Eq. (77) above because of spherical curvatures introduced for cells, for permeability considerations, and other special complexities applicable to cell metabolism. The theoretical expressions were fitted with considerable success to various experimental data from many authors on respiration of sea urchin eggs, luminescent bacteria, and *Chlorella* cells.

For purposes of recognizing general systems functions, we indicate again that the asymptotic form of all of these graphs is due not to peculiarities of cell metabolism, but will result from any double flux situation. The form of an asymptotic limiting factor graph is given in Fig. 10c, and the effect of varying both flows is shown on the quadratic surface. As one increases the limiting factor, the reaction output approaches an asymptote at which the other flow is then limiting.

The equations for double flow interactions with limiting aspects resemble those of the Michaelis–Menten kinetics of the cycling receptor module but there is a fundamental difference since no recycling materials are required for the work gate performance as a limiting hyperbola.

F. Work Gates As Valves

Before they meet one of the two multiplicative flows clearly carries energy, whereas the second flow, also essential, seems to involve relatively small flows of mass or energy. Regardless of its weight or energy contribution, the limiting and controlling flow is the one inflowing without excess, N_1. Examples are trace elements required for photosynthesis and relay circuits in power stations. In such systems the small flow serves as a gate controlling large energies. Complex environmental systems have their power circuits modulated by control systems that operate gates, some of which are multiplicative double flux junctions. Double flux junctions are network valves.

There are two energy values which may characterize the limiting flow: one its energy contribution to the module before reaction and the other the amplified value of its energy role.

Sugita (1961, 1963) and Sugita and Fukuda (1963), discussing enzyme–chemical circuits, term the regulating flow at a double flux junction a "throttling factor." An enzyme participating in a reaction and being regenerated does follow the Michaelis–Menten relation which is a special case of a double flux junction involving feedback as considered below with Fig. 11. Sugita discusses negative and positive catalyzers. Auxiliary flows are compared to logic circuits where go or no-go throttles are operated by one circuit upon another. In the macroscopic world, both on–off regulators and varying-rate regulators occur. A

limiting and controlling flux can serve as a rate-controlling valve; but if action thresholds are involved, off–on valve actions may result.

XI. The Switch Module

Logic circuits are a special case where work done by one or more energy circuits controls another with only on and off positions. The energy symbol for on–off valves is in Fig. 1i. A valve is a double flux multiplicative junction with one of the two flows regarded as the control on the conductivity on the other.

Figure 1i is the general energy switch symbol used for all digital logic functions. The program of output response to input energy flows and forces must be specified in detail. These switches may serve as *and*, *nand*, *or*, or *nor* gates, etc.

On and off switching may represent birth and death of pathways of organisms. The switch symbol, for example, may show the convergence of work energy of one male and one female, the output being proportional to effective coincidence of the two input flows (forces), the output being reproduction. Another example is the requirement which controls converging voting work actions of people in elections. A logic function for a system of two individuals becomes a finite population transformer as in population genetics.

The switch requires maintenance energy by at least one of the inflowing processes and like other complex modules the switch function has a heat drain associated with its pathway maintenance.

XII. Self-Maintaining Module

Figure 1g shows the self-maintaining module. In its simplest form there is potential energy input being in part restored against potential generating forces, and then routed to increase flow from the upstream source by at least one kind of work on the upstream flow. Cells, organisms, populations, cities, and other units with respiration may be represented by modules of this class although most involve many kinds of storage and self-affecting work. *To completely specify the characteristics of even the simplest module requires the storages to be characterized by its storage-to-force ratio [(C in Eq. (21)], by its work gate module [k in Eqs. (63)–(65)]; and by the ratio of potential generating work to dissipational flows.*

The growth of the storage with this arrangement is logistic. Component [illegible] are given in Fig. 11, which also shows variations [illegible] of several positive feedback loops.

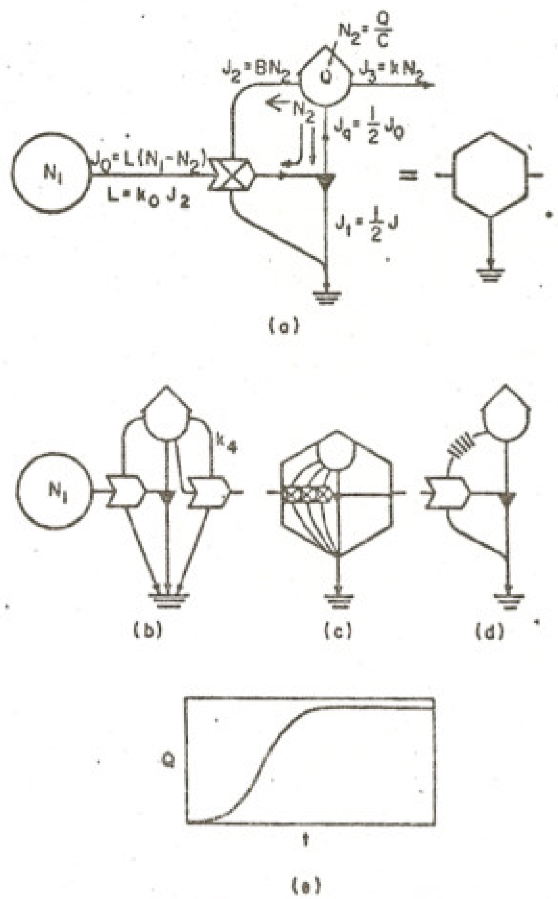

FIG. 11. Component processes of the self-maintaining module with logistic growth form.

FEEDBACK LOOPS WHICH MULTIPLY

When a feedback intersection involves a multiplicative reaction with the input shown in Fig. 11, the multiplicative loop is sometimes said to be autocatalytic by chemists and logistic by population biologists. To obtain an equation for the loop transfer function, one writes expressions for each arm of the circuit shown (Fig. 11) and combines them. The input force N_1 minus the backforce from N_2 is given by

$$J_0 = L(N_1 - N_2). \tag{80}$$

The effect on the feedback flow is

$$J_2 = BN_2, \qquad L = k_0 J_2. \tag{81}$$

At maximum power adjustment J_q is half J_0:

$$J_q = J_0/2. \tag{82}$$

The down-circuit drive is given by

$$J_3 = kN_2. \tag{83}$$

The quantity being stored Q delivers force to its pathways according to Eq. (21). The rate of change of the storage Q is the balance between the inflow to storage J_q and the outflows from the storage to feedback J_2 and downcircuit J_3:

$$dQ/dt = J_q - J_2 - J_3. \tag{84}$$

Substituting in (84) from (81), (82), and (83) one obtains a differential equation, (85), which can be recognized as logistic by comparing with Eq. (87) when N_1 is constant:

$$\frac{dQ}{dt} = \frac{1}{C}\left[\frac{k_0 BN_1}{2} - B - k\right]Q - \frac{k_0 B}{2C^2}Q^2. \tag{85}$$

Since the export flux J_3 is in proportion to the storage force, the transfer function for the multiplicative loop is also logistic. Setting Eq. (85) equal to zero and solving for Q, one obtains the steady-state carrying capacity as an energy-dependent expression, $CN_1 - [2C(B + k)]/k_0 B$.

The logistic equation has been much used for study of the macroscopic world and its usual form in ecology [Eq. (87)] may be compared with Eq. (85). If the magnitude of the constant percent change rate is linearly and negatively diminished as the quantity q approaches a limiting value ($q = K$), then the differential equation, (86), results, the logistic equation:

$$dq/q\,dt = k(K - q)/K. \tag{86}$$

Sometimes it is useful to rewrite Eq. (86) in the form

$$dq/dt = kq - (kq^2/K). \tag{87}$$

For Eq. (87) a verbal statement can be made that the rate of change of the quantity q is proportional to the quantity q [illegible] proportion to the chance of interactions q^2. If there is no backforce, the module may still operate by logistic form if the outflows are being pumped out by one interior circuit pumping or gating another as shown in Fig. 11b. Energy draining interactions may be proportional to mass action and hence to the square of the number of population units. If there are neither backforce or forward force interactions that produce negative square terms and the energy source is of constant force type, then the module may grow exponentially (Malthusian). If the energy source is of constant flow type, the growth reaches an asymptote determined by the level of input energy in a Von Bertalanffy charge pattern.

Integrating both sides of (86) with respect to time, one obtains the exponential form, Eq. (88) where $a = \ln(K - q_0/q_0)$ and q_0 is the starting storage, when $t = 0$:

$$q = K/(1 + \exp a - kt). \tag{88}$$

An s-shaped curve results, shown in Fig. 11, in which the value of the quantity q levels off at K. This equation has often been used to represent growth of a population of animals under the simplifying conditions stated above. One set of experiments whose behavior fits these simplifications is given by Ikusima (1962), working with duckweed.

The form of the logistic equations in Eqs. (86) and (87) has been related to biological premises such as an intrinsic rate of natural increase, implying that k can be constant for a population. The derivations in Eqs. (80)–(85) show the same type of equation emerging from different component assumptions about circuits, forces, and flux which provides a more general interpretation. The percent growth when storage is small is a function of the energy source and input force and is not really intrinsic. The ecological forms of the equation [(86) and (87)] are special cases of the more general circuit (Figs. 9g and 11).

These derivations illustrate the many response possibilities of the simplest self-maintaining circuits, which are relatively easily represented in circuit language but become cumbersome as differential equations. Most real self-maintaining modules (organisms, populations, human groups, etc.) have much more complexity in internal programs of time lag, multiple loopbacks, multiple storages, and packaged subroutines. It remains to be seen how much error there is in visualizing such complex modules as though they had the performance of the simplest case. Ultimately any features of the real module which vary should be diagrammed as part of the energy network representation and analysis.

XIII. Cycling Receptor Module

Shown in Fig. 1e is the module which includes a feedback of downstream energy interacting multiplicatively and positively with the upstream flow but with the additional constraint of being dependant on the recycling of a material which is constant over short periods. The receipt of light energy by an electrical photocell, by photosynthesis, and other processes is included as shown in Fig. 12. The kinetics of this module are also given in Fig. 12, which has an equation for output as a hyperbolic function of the input, the output leveling off as the recycling process becomes limiting.

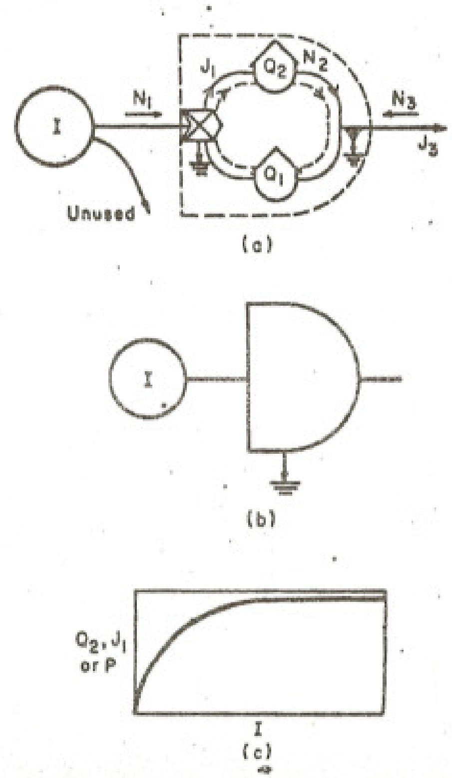

FIG. 12. Cycling receptor module and component processes.

Receptor circles are an important class of circular systems including closed material cycles that receive inflows of pure energy, transforming and storing them often as the first stage of other complex networks. The recycling material reacts with the energy flow to form an activated energy-rich state, that is stored temporarily as a potential. Then the receptor is returned to its receptive state, the potential energy may drive other processes down-circuit. The sketches in Figs. 12 and 14 illustrate operation of receptor circles. Balls falling into a bowl or water waves on a rocky shore splash water from the low energy state into an elevated reservoir from which it may flow down again doing work. The receptor material is water.

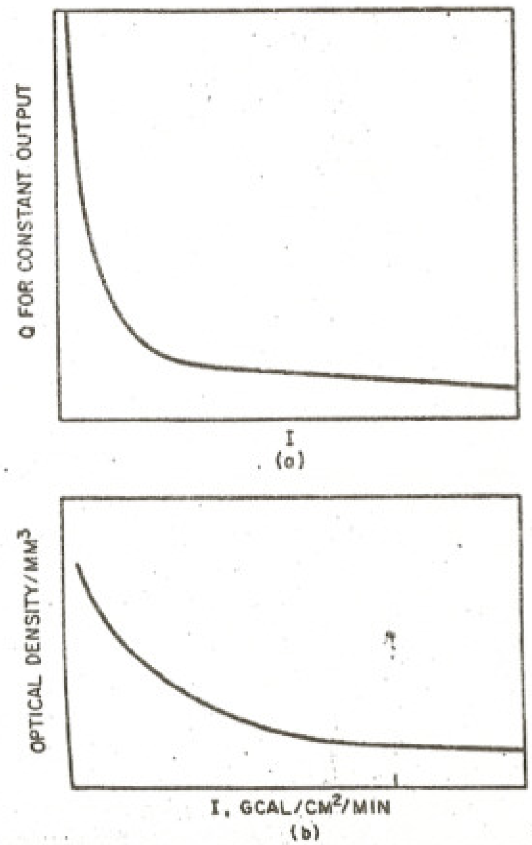

FIG. 13. Quantity of cycling receptor for delivery of constant output load with force varying. (a) Theoretical prediction of cycling receptor module, $Q = (K + kI)I$; (b) chlorophyll as a function of light intensity in stacked dishes of *Chlorella* cultures (Phillips and Myers, 1954).

The energy and material circuits (dashes) for receptor circles are indicated as (c) in Fig. 12. Since the feedback junction involves reactive [illegible]

as indicated by the × symbol. The loop arrangement tends to smooth out variation. When energy inflows are excessive, the receptor is kept in the upper reservoir, unavailable for further input reaction; but when inflows of energy are small, most of the receptor is back in receptive state increasing the probability of reaction.

Although quite different examples are given in Fig. 14, the energy circuits are similar. In (a) ocean waves throw water on an elevated reef from which it flows back to the sea level. In that example the receptor water is unlimited. In (b)–(d) electrons are the recycling material, receiving sound and light energies; (d) has pulsed electrical flow in an input circuit storing energy in a second circuit by induction. In (e) mercury vapor is the receptor, and in (f) the revolving cups of the wheel are the receptor. The Michaelis–Menten formulation for an enzyme-substrate reaction is shown in (g). Most interesting of all are the photosynthetic receptor systems involving chlorophyll. For simplicity all the loads drawn in Fig. 14 are dissipative drains although other kinds may be substituted.

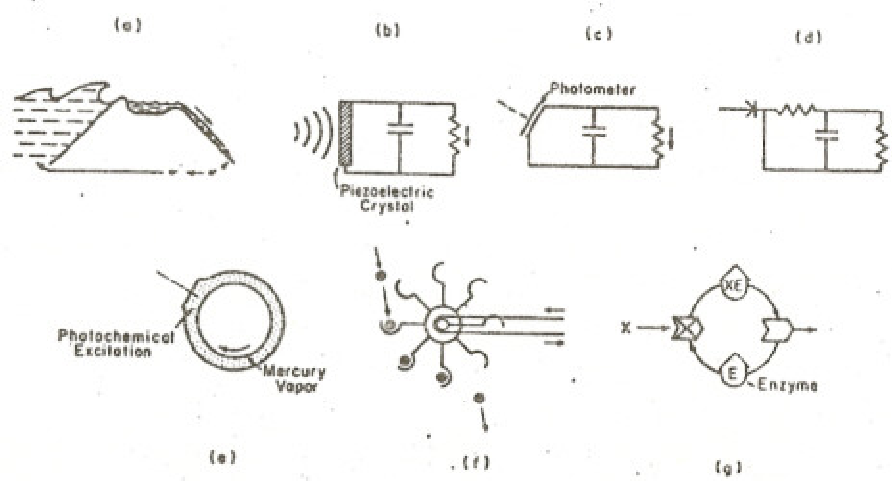

FIG. 14. Examples of cycling receptor systems receiving trains of pulsed energy. (a) water waves, (b) sound, (c) light, (d) pulsed voltage, (e) light, (f) cannon balls, (g) substrate packages.

Cycling receptor systems convert pure energy flows like sound, light, and other wave trains into energy storages associated with matter. Pulsed energies with matter also follow this scheme. As the kinetic [illegible] shows, the circular aspect [illegible]

A. Kinetics of Receptor Circles

The kinetic behavior of a cycling receptor circle is given next in a general way applicable to all of the examples cited. Derivations for particular examples were independently given long ago. For example, Michaelis and Menten (1913) and Hearon (1949) provided expressions for enzyme reactions, Mitchell and Zemansky (1934) gave equations for light reception by mercury vapor, Rose (1961) expressed the photochemical activation of anthracene, and Lumry and Reiske (1959) provided formulations for chloroplasts. Consider the notations written for the energy diagram in Fig. 12.

Energy is inflowing past the receptor zone with intensity I so that N_1, the concentration of energy packets in the range of reception, is in proportion:

$$N_1 = k_0 I. \tag{89}$$

The total quantity of receptor material is the sum of that in the receptive low energy state Q_1 and that in high energy storage reservoir Q_2:

$$Q = Q_1 + Q_2. \tag{90}$$

The multiplicative reaction of the energy and the receptor material produces a production reaction flux J_1 carrying activated receptor and energy into the storage reservoir marked Q_2:

$$J_1 = k_1 N_1 Q_1. \tag{91}$$

If the force exerted by the storage is in proportion to the amount of receptor activated there, then

$$N_2 = Q_2/C. \tag{92}$$

Flows out of the reservoir go in proportion to the number and force of the activated receptor units there. Hence the recycling flow of receptor J_2 is in proportion to N_2. It is a dissipative work:

$$J_2 = LN_2 = (L/C)Q_2. \tag{93}$$

For a particular inflow condition a steady state flux J may be established when the flow into the reservoir J_1 equals the recycling flow J_2. Setting (91) equal to (93) and substituting (89) and (90) results in

$$Q_2 = Ck_0k_1QI/(L + Ck_0k_1I). \tag{94}$$

The population force N_2 from the upper potential energy storage is obtained by combining Eqs. (92) and (94):

$$N_2 = k_0k_1QI/(L + Ck_0k_1I). \tag{95}$$

The steady-state flux J in the cycling material is obtained from Eqs. (93) and (95):

$$J = Lk_0k_1QI/(L + Ck_0k_1I). \tag{96}$$

For systems which have energy storage in linear proportion to flux, as in many biological storage processes, power delivery P into productive storage compartment is proportional to potential energy value per unit flux μ:

$$P = \mu Lk_0k_1QI/(L + Ck_0k_1I). \tag{97}$$

The form of this expression is shown in Fig. 12, the familiar graph for photosynthesis with light or enzyme reaction as a function of input fuel substrate.

If there is an output circuit with loading backforce N_3 driven by energy in Q_2, the output flux is the sum of force N_2 from the reservoir and load, the latter being negative (see Fig. 12a):

$$J_3 = L_{23}N_2 + L_{33}N_3. \tag{98}$$

Combining (95) and (98) one obtains the output in terms of the energy inflow to the receptor circle:

$$J_3 = \frac{L_{23}k_0k_1QI}{L + Ck_0k_1I} + L_{33}N_3. \tag{99}$$

Rose (1961) provided a procedure for analyzing data for such hyperbolic relations [Eqs. (94)–(98)] plotting reciprocals to obtain a straight line. Thus, the storage potential N_2 and the pertinent fluxes J and J_3 of a receptor circle vary with the input energy flow I according to rectangular hyperbolas like that plotted in Fig. 12. This relation is for steady state.

Passive or operational electrical analogs can be arranged. The photometer in Fig. 14 is a special case and may also serve as a passive analog. All these relationships define the symbol in Fig. 12b.

B. Power and Efficiency of Receptor Circles

Figure 7 shows the efficiencies and power properties of a simple transformation system in which the output load X_2 is held constant and the input load X_1 varied. Except for one particular zone of loading [illegible] by a circle, the transformation was either very inefficient or [illegible] not drawing much power. In a regime of varying input force, the system would rarely be set at the optimum loading for maximum power transmission of potential.

In comparison, consider the graph of the hyperbolic transfer function (Fig. 12c) for the same variables and coordinates. The presence of a cycling receptor Q which is limited in quantity causes higher efficiencies by allowing excess energy to pass without reception for use by other units. The cycling receptor serves as a feedback governor on the loading. Energy bypassed is available for use in other units. Both P_1 and P_2 follow the shape of the graph in Fig. 12.

C. Loading and Power Transmission Optima in Photosynthesis

The input–output loading arrangements in the cycling receptor systems of Figs. 11 and 12 at steady state permit the same kind of rate variations with loads as discussed for potential generation work modules. For maximum conversion of energy into storage potential there is a particular loading ratio. If the input light is held constant so that the density of photons available to the receptor sweep is constant, the input force is also constant [Eq. 89]. By varying the output load from zero to a stopping load one obtains the hump-shaped output curves already given for other systems in Fig. 5. Photosynthetic examples were represented (Odum, 1968) where data from Lumry and Spikes (1957) and Clendenning and Ehrmantraut (1951) for the Hill reaction with varying concentrations of reactants were graphed in a hump shape. A blue-green algal mat under constant light provides a hump-shaped power delivery with varying external loading (Armstrong and Odum, 1964). The receptor system follows the optimum efficiency-maximum power principle.

However, in most photosynthetic situations the input varies diurnally and seasonally with the march of the sun. For optimal loading to be maintained in spite of varying inputs, some flexible ways of changing the loading with input are required. Gears have to be changed as in the accelerating car. Whereas a simple potential-generating transformation has a very poor response under such conditions with a poor setting most of the time, as shown in Fig. 7, the cycling receptor system has a better regulated response (Fig. 12).

The single cycling receptor system draws less power and has a higher average efficiency under the varying input regime. Systems of several consecutive loops have even more stable patterns, as discussed by Heinmets and Herschman (1962). Plants have additional special mechanisms for maintaining favorable loading.

D. Changes in Pigment Cycling Systems for Load Adaptation

A number of mechanisms are known which help to adapt the plant receptor system to its loading. Some are rapid responses characteristic of the loop; some are adaptive physiological responses involving additional special circuits; and some are adaptations accomplished by substitution of species in the system. The coupling of flows from one pigment to another as drawn by French and Fork (1961) provides means for stabilizing output where wavelengths are varying.

The adaptive properties of the hyperbolic loop function can be recognized by study of Eq. (99) for downcircuit flow. When light input is small, Ck_0k_1I is small so that the relation approaches that in Eq. (100):

$$\underset{(I\ \text{small})}{J_3} = k_0k_1QI + L_{33}N_3 . \tag{100}$$

When light intensities are high, Eq. (100) approaches Eq. (101), the asymptote of the hyperbola:

$$\underset{(I\ \text{large})}{J_3} = (L_{23}Q/C) + L_{33}N_3 . \tag{101}$$

When Eq. (99) is solved for Q, the quantity of total receptor cycling, Eq. (102) results:

$$Q = (k_0k_1CI + L)(J_3 - L_{33}N_3)/L_{23}k_0k_1I. \tag{102}$$

At any light intensity the output may be increased by augmenting the amount of chlorophyll and other pigments that can be brought into action. Limits to adding pigment are the physical limits of space for more pigment, the energetic costs of synthesis and maintenance, and the need to match input to the output loadings.

From Eq. (102) Q, the pigment required to maintain a constant output of photosynthesis J_3 at constant load N_3, is in inverse relation to the light intensity. As graphed in Fig. 13, the drive applied to downstream loading can be adapted to increases in light intensity by diminishing the pigment. The effect is large only at small light intensities. Shown also in Fig. 14 is a graph of experimental data for *Chlorella* from Phillips and Myers (1954). Wassink *et al.* (1956) report such adaptations in maple leaves, Gessner (1937) in land and water plants, and Steeman-Nielsen (1962) in plankton, to name a few.

XIV. Production and Regeneration Module (P–R)

Given in Fig. 15 is the module formed by combining a cycling receptor module, a self-maintaining module which it feeds, and [illegible] feedback loop which controls the inflow process by multiplicative and limiting actions. An example is the green plant, which has a respiratory system. More complex examples have more than one respiratory system (Fig. 15). An example is the plant and animal symbiosis of corals and zoochlorellae-containing hydra. On a large scale the module represents plants and consumers of ecosystems or agriculture and cities. It was already shown that the cycling receptor module had a hyperbolic transfer function due to the limits that develop by necessary internal cycles. The P–R module adds more cycles. Generally, the form of the output of a group of cycling processes coupled into one system behaves with an asymptotic pattern not unlike that of the simpler single recycling system. For example, analog simulation of models in Figs. 15–17 show this stability.

XV. Economic Transactor Module

For human systems with circulating money currency, Eq. (103) and the module in Fig. 1j pertains:

$$J_{\text{energy}} = -kJ_{\text{currency}} . \qquad (103)$$

Flows of currency move opposite in direction to the flow of potential energy and work but are at each transactor point regulated to go in proportion to the price established for the goods or services. The money flow must balance and this happens if there are loopbacks of valuable work services that have upstream amplifying effects that are worth as much money as those with more power. The money system ensures some distribution of closed loops of service organizing each compartment of the system in exchange for return payments and energy flows. Shown in Fig. 16 are some usages of the economic transactor system.

The relationship of energy flow and money flow shows that in a macroeconomic sense, the energy and money flows are coupled and neither can be understood without the other. Since there is a balance of work services of comparable value among components and a balance of money payments it is possible to determine the ratio of one to the other in a given economy at a given time by dividing the total energy budget by the total money budget. This ratio allows one to convert the dollar value to the energy value at its loop amplified maximum.

Control circuits thus have two energy values, their own and that which they obtain in their work gate amplification. The relationships of a simple economic cycle are given in Fig. 16, which expresses the relation- [illegible]

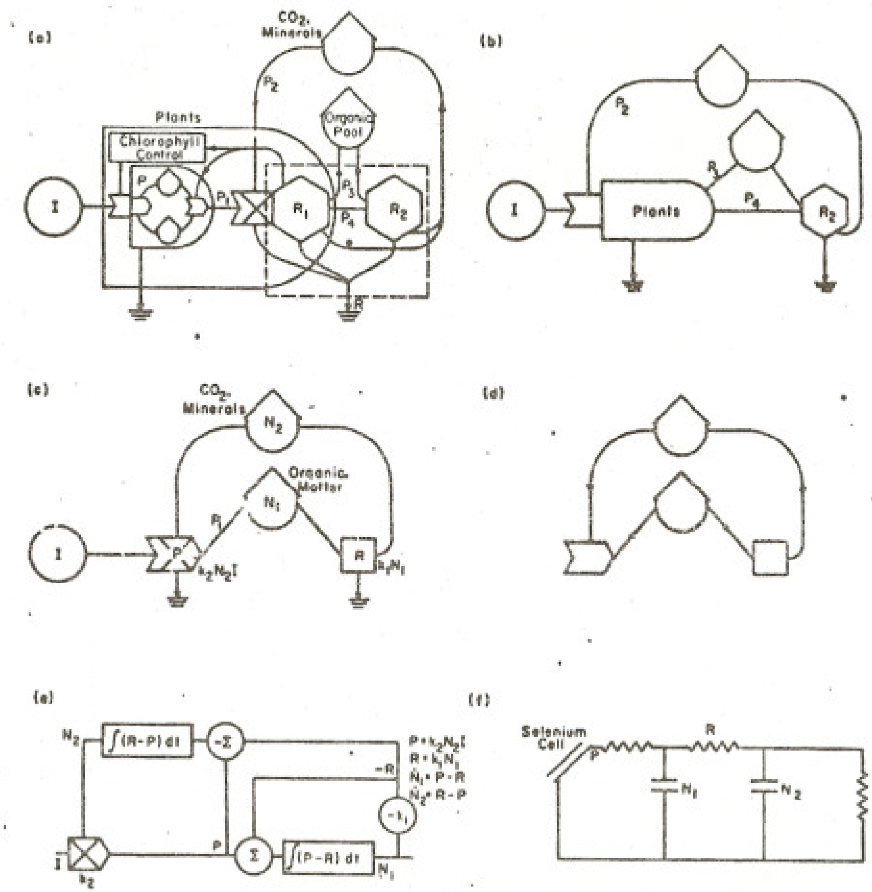

FIG. 15. System of production and consumption in energy circuit language illustrating cycles of work feedback loops within cycles. The process of modeling, simplification, and simulation is illustrated in the sequence from (a)–(e) or (f). (a) Energy #1. An ecological system showing plants and consumers doing respiration and maintenance *R*. Within plants is a cycling receptor module, a workgate receiving control loop from mineral-CO_2 mix pool, and the plant's own respiratory system R_1. Total respiration in the dashed box includes plant and animal–microbe respiration units which feed by-products into nutrient-CO_2 pool. (b) *Energy #2. Diagram (a) is further abbreviated by eliminating detail within the plant (P–R) module.* (c) *Energy #3. Diagram (a) simplified further into two von Bertalanffy storage modules, one work gate control of mineral feedback isolating the essence of a productive and respiratory process of the ecosystem as measured when studies are made of photosynthesis and respiration of microcosms.* (d) *Mineral cycle in simplified model.* (e) *Block diagram of differential equations showing the double loop of negative feedback* [illegible]

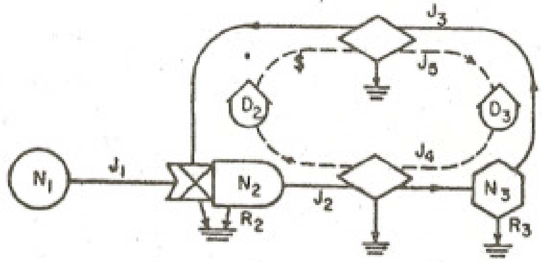

FIG. 16. A system of recycling currency showing component processes and modules, where N_2 is the agriculture sector and D_2 its capital; N_3 is the urban sector and D_3 its capital.

The ratio of energy flow to currency flow is maximum near the source of energy decreasing downstream, but the energy value as amplifier control increases downstream in any surviving system which has its downstream units reward-looped to the upstream inflow modules. The energy definition of value of a flow is the sum of the two energy components (actual flow and amplified loop effect). If the downstream units are people, their salaries are highest in proportion to their amplified loop effects, since their actual power consumption is small and little varying. Macroeconomics is a system of currency and energy, the full analysis of which is not possible using one of the component networks alone. The diagram for a simple two-module simplification of an economy with agricultural and city sectors in Fig. 16 implies the following equations:

Work gate on agriculture:

$$J_1 = k_0 N_1 J_3 ; \qquad (104)$$

Rate of population force accumulation in agricultural sector ($N_2 = Q_2/C_2$):

$$\frac{dQ_2}{dt} = \frac{L k_1 Q J_1}{L + k_1 C J_1} - J_2 - R_2 ; \qquad (105)$$

Price control modules:

$$J_2 = k_2 J_4 , \qquad J_3 = k_3 J_5 ; \qquad (106)$$

Rate of population force accumulation in city sector:

$$dQ_3/dt = J_2 - J_3 - R_3 ; \qquad (107)$$

Currency flows from capital storages D:

$$J_4 = k_4 D_2 , \qquad J_5 = k_5 D_3 ; \qquad (108)$$

Rates of change of capital:

$$dD_2/dt = J_3 - J_5 , \qquad dD_3/dt = J_5 - J_4 ; \qquad (109)$$

Constant total currency in system:

$$D_2 + D_3 = K; \qquad (110)$$

Energy costs of maintenance:

$$R_2 = k_6 N_2 , \qquad R_3 = k_7 N_3 . \qquad (111)$$

On analog, the model's behavior depends on functions used to control prices k_2 and k_3.

XVI. System Examples and Their Simulation

Some energy circuits with various degrees of complexity are given in Figs. 15–17. Included are some for very simple physical processes and others for systems of man and nature. The reader will recognize various degrees of combining and compartmentalizing of detail, a practice which hides detail and creates a model to help the human mind visualize broad features of the system and its possible responses.

For simulation, modules of the energy language carry with them that characteristic performance as often written with differential equations. The energy diagrams are themselves mathematical expressions of the network or our model view of it. It is almost an automatic process to substitute difference equations or differential equations for each module. Then analog or digital means are used to keep a running computation of the stocks, flows, and rates of the many parts of the system under testing with various forcing functions and other experimental manipulations that answer questions about the system's performance. When the temporal responses of the model have similarity with that of the measured system, one is encouraged that there is truth in the model and further details are added and tests are made to determine if the similarities are not fortuitous. An example of this process is the balanced ecosystem studies in small chambers containing rain forest soil, litter, herbs, small animals, and microorganisms. The model is given in Fig. 15c–f. When simulated by Odum et al. (1970), graph properties were obtained which were similar to the results of [illegible]

The coefficients were evaluated by substituting known values of flux when storages were known. The steady-state annual flows and fluxes may be used for this purpose as a first approximation. It is often convenient to write steady-state values of stock and flux on the diagram as in the example, Fig. 17. The outflow coefficients are the ratios of flux to stock, the fraction passed per unit time (see k in Fig. 6). The reciprocal is the time constant, the steady state turnover time.

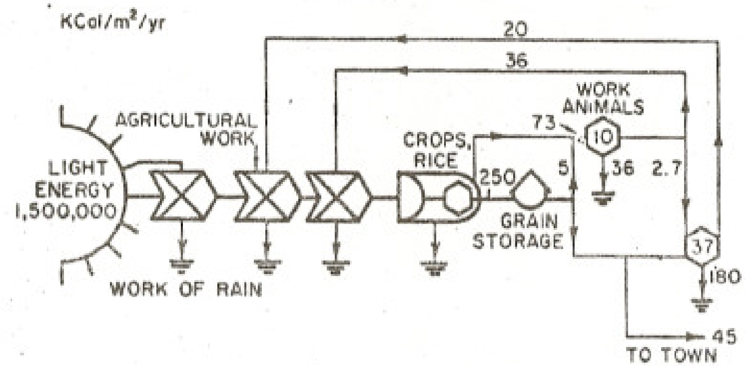

FIG. 17. Example of a system of man and nature in energy circuit language with stocks and steady-state flows indicated (Odum, 1968). Monsoon agriculture in India is portrayed showing the role of sacred cows.

If passive analog simulation is used, the energy circuit language has its equivalent circuits (see example in Fig. 15f). If operational analog simulation procedure is to be done, the block diagramming of the integral equations are drawn as in Fig. 15e which also shows where there are properties of feedback stabilization. If digital simulation is to be done, the difference equations for each module are written so as to add the changes for an increment of time followed by a print and loop statement. For the example in Fig. 15c, the digital program for varying light I in English becomes:

1. INPUT $N_1, N_2, N_3, k_1, k_2, I, P, R, X, Y, \dot{N}$ as memory locations,
2. INPUT initial conditions for N_1, N_2, I, k_1, k_2, T,
3. PRINT N_1, N_2, I, k_1, k_2, T,
4. Let $P = k_2 N_2 I$,
5. Let $R = k_1 N_2$,
6. Let $X = N_1$,
7. Let $N_1 = N_1 + P - R$,
8. Let $N = N_2 - X$,

9. Let $Y = N_2$,
10. Let $N_2 = N_2 + R - P$,
11. Let $\dot{N} = N_2 - Y$,
12. Let $T = T + 1$,
13. PRINT OUT $N_1, N_2, \dot{N}, P, R$,
14. If T has reached desired stopping place, stop; if not, continue.
15. Return to Statement 4 and repeat with a new input value for I.

Thus the simulation procedure for the energy circuit follows in simple automatic manner from the energy circuit diagram; the thinking on the behavior and structure of the system is done in the diagramming. Since groups of simulation and system characteristics go with each module, the modules are the words of the language which carry group laws and temporal transient characteristics that are helpful in thinking and expression in the same manner as in other languages. Many kinds of systems from the physical and molecular to the ecological and social can be expressed by the same language and the diagramming helps us to recognize many as special cases of a relatively few general system types. The examples used here to illustrate the modules of the energy circuit were single processes that defined the behavior of modules in their simplest form in terms of well-established kinetics and energy laws. In application to more complex systems one recognizes the type of module in the system before the exact kinetic aspects are known. For example, populations of people, cities, land divisions, etc. are self-maintaining modules of the class of the logistic module although complexity in these units makes it unlikely that the module would have an overall behavior exactly that of the simple, single, linear, feedback multiplication. The details would have to be verified by performance studies and diagramming of the within-module circuits. However, recognizing the class of the module does carry some semiquantitative feeling about the nature of the growth, stabilization, and power processing. In time the behavior of more and more configurations will become known, and the meaning to the language reader will also increase with his knowledge.

Offered as a general systems language, the energy language allows all kinds of factors to be included in the same network. One may keep track of the budgets of flowing quantities while at the same time dealing with dynamic aspects such as forces, differential equations, and transients. It attempts to connect the biological and economic worlds to those of electronics and systems science. [illegible] arguments and is efficient in making ideas clear; at best it leads readily from the real world data to realistic computer simulation.

The language has the interesting property of showing many entirely different kinds of systems as similar in type. For example, food chains resemble eddy diffusion chains; hurricanes resemble organisms; war functions resemble carnivore actions on two competing food populations.

Hopefully the language synthesizes many disciplines. For example, the work gate brings in physiological ecology; the cycling receptor module, biochemistry; the configurations of one or two self-maintaining modules, population ecology; the reward loops, mineral cycles; the numerical flows, ecological energetics; the transaction module, economics; the analysis of the diagrams produced, matrix ecology; and the more elaborate control modules, the switch; and combinations, behavioral sciences.

Energy diagrams for rain forest and radiation processes are given in our rain forest book (Odum and Pigeon, 1970), for marine systems in our coastal ecosystems report (Odum *et al.*, 1969), and for photoregeneration, in Odum *et al.* 1970b. For previous applications, see Odum (1967a, b, 1968), and for a general account, Odum (1970). I am grateful for comment from David Cowan, Gettysburg College.

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